Abstract

We study the impact of tariffs on the margins of homogeneous-input trade and examine Home optimal tariffs in a vertical oligopoly model where Home specializes in final goods and Foreign specializes in intermediate inputs. In the exogenous (endogenous) market structure, we find that (i) Home tariff reductions raise Foreign exports mainly through increases in the intensive (extensive) margin, and (ii) Home optimal tariffs are higher, the thicker (thinner) the Home final-good market relative to the Foreign intermediate-input market. To assess the empirical relevance, we exploit vertical linkages between Japan and China. Our estimation results are consistent with predictions in the endogenous market structure.

Keywords: Tariffs, Vertical Oligopoly, Free Entry, Extensive Margin, Intensive Margin

JEL Classification Numbers: F12, F13
1 Introduction

Recent years have witnessed faster growth in intermediate-input trade than final-good trade. It is often argued that rapid growth in intermediate inputs has triggered by vertical specialization, which allows each country to provide only a particular stage of final-good production sequences by fragmenting production processes across the globe (Yi, 2003, 2010). It is also often argued that not only does foreign direct investment but also foreign outsourcing plays a key role in vertical specialization (Hummels et al. 2001; Hanson et al., 2005). When domestic final-good producers outsource some intermediate inputs to foreign intermediate-input suppliers under contractual agreements, the vertical relationship of this kind is frequently captured by a bilateral oligopoly model with bargaining over the terms of contracts that specify the input price and its quantity. In practice, however, a large fraction of intermediate inputs are internationally traded through spot markets among anonymous final-good producers and intermediate-input suppliers rather than contractual agreements that are made between matched pairs.

There is established evidence that documents the difference between contractual agreements and spot markets in the international trade literature. Rauch (1999) divides manufactured final goods that are internationally traded into three groups (sold on organized exchanges, reference priced or neither) and finds that differentiated final goods tend to be internationally traded by proximity and preexisting ties (such as contractual agreements), while homogeneous final goods tend to be internationally traded through spot markets. Applying Rauch’s (1999) classification to intermediate inputs, Nunn (2007) also finds that differentiated intermediate inputs tend to be exchanged by non-market mechanisms (such as contractual agreements), while homogeneous intermediate inputs tend to be exchanged through spot markets. In addition to this fact, there is another kind of evidence that countries involved in vertical specialization – especially China – tend to export homogeneous intermediate inputs. For example, adapting Schott’s (2008) export similarity index to vertical specialization, Dean et al. (2011) find that the similarity between Chinese and OECD intermediate-input exports is generally very low. Dai et al. (2016) also find that Chinese intermediate-input exports in vertical specialization (i.e., processing exports) are less skill and R&D intensive. This evidence suggests that a large fraction of intermediate inputs produced in vertical specialization should be homogeneous in nature and, together with the first evidence, they should be internationally traded through spot markets.

Building on these pieces of evidence that contractual agreements are not the only means of procuring intermediate inputs in vertical specialization, we develop a bilateral oligopoly model in which market-based interactions between vertically related industries have a crucial impact on trade policy. Following the evidence that countries involved in vertical specialization provide only a particular stage of production sequences, we assume that one country (Home) specialized in producing final goods whereas another country (Foreign) specializes in producing intermediate inputs. Further, following the evidence that homogeneous goods are exchanged in spot markets, we assume that these countries produce homogeneous goods and the price of final goods (resp.
intermediate inputs) is determined at the market-clearing levels in the Home (resp. Foreign) market. A Home government imposes a tariff on Foreign intermediate inputs to maximize its welfare, even though final-good production in Home relies on intermediate inputs imported from Foreign. In this setting, we derive the optimal tariff in different market structures where the numbers of firms in the markets are either exogenous or endogenous. In so doing, we show that different market structures have different impacts of tariffs on the number of firms (extensive margin) and the average output per firm (intensive margin) of homogeneous goods produced in vertical specialization, which in turn affects the characterization of optimal tariffs.

In the exogenous market structure where the number of firms is fixed and only the quantity is responsible to trade policy, we find that the Home optimal tariff is higher, the thicker is the Home final-good market (relative to the Foreign intermediate-input market). An increase in the Home tariff rate leads to a less-than-proportionate increase in the price of intermediate inputs for all demand functions that are strictly logconcave. Since Home imports intermediate inputs from Foreign, this works as a terms-of-trade improvement. Counteracting this welfare gain is a welfare loss due to a tariff-induced reduction in the quantity of each firm. The strength of these two forces that characterize the optimal tariff is influenced by market thickness. Suppose that the number of Home firms is arbitrary large that the Home market is perfectly competitive. If the Home government faces perfectly competitive domestic market, the rationale of imposing a tariff is only the terms-of-trade gain at the expense of Foreign, which induces the Home government to impose a positive tariff. Suppose another extreme case where the number of Foreign firms is arbitrary large that the Foreign market is perfectly competitive. Then, the situation is like a single-stage Cournot oligopoly in Home, in which case a positive subsidy (i.e., a negative tariff) increases Home welfare by narrowing the wedge between the price and marginal cost. As the number of Home firms is greater and the Home market is thicker, the reduction in the quantity of each firm is smaller relative to the terms-of-trade improvement, and the higher tariff is more likely to be optimal for Home. This yields a monotone relationship between the optimal tariff and market thickness in that the optimal tariff is higher, the more competitive the Home final-good market relative to the Foreign intermediate-input market.

An interesting by-product of our analysis is the possibility of a positive relationship between competitiveness of the Foreign market and Foreign profits. Because of the induced lower tariffs, we find that it is possible that an increase in the number of Foreign firms not only benefits Home firms but it can also raise the profits of Foreign firms. This counter-intuitive outcome is more likely when the number of Foreign firms is smaller or the curvature of the inverse demand of final goods is bigger.

In the endogenous market structure where the number of firms is also variant to trade policy, we find that this relationship is overturned and the Home optimal tariff is higher, the thinner is either the Home market or the Foreign market. As in the exogenous market structure, the optimal tariff is dictated by the reduction in the quantity and the terms-of-trade improvement. In the endogenous market structure, however, these two forces do not always occur for all log-
concave demand functions, since in addition to directly increasing prices, a tariff also indirectly increases prices by reducing the number of firms. By virtue of this additional adjustment, we show that the sign of the optimal tariff depends on the elasticity of demand and the optimal tariff is positive (negative) for strictly concave (convex) demand. In this setup with free entry where the number of firms is endogenous, the key exogenous parameter that shapes market thickness is the entry cost. Suppose that Home firms face the lower entry cost and find it easier to start a business in Home, which makes the Home market thicker than the Foreign market. Then, in vertical specialization where Home output and Foreign input are complements, this encourages not only entry of Home firms but also entry of Foreign firms into the respective market. Further, since both markets are more competitive, it is optimal for Home to adopt freer trade, the lower is the Home entry cost. The similar claim holds when Foreign firms face the lower entry cost. Thus the Home government is prompted to set lower tariffs if either the Home market or the Foreign market is more competitive. This yields a non-monotone relationship between the optimal tariff and market thickness in that the optimal tariff is higher, the less competitive both the Home final-good market and the Foreign intermediate-input markets.

It is not surprising that different market structures give different predictions, and it is an empirical question to ask which market structure is more apt in reality. To assess empirically the relevance of our predictions in different market structures, we construct a unique industry-level trade dataset by noting a vertical linkage between Japan (Home) and China (Foreign). The first prediction is concerned with the response of the extensive and intensive margins to tariff changes. Our theory predicts that, although a reduction in Home tariffs increases the Foreign aggregate exports in both market structures, this is largely accounted for by the increase in the intensive (extensive) margin in the exogenous (endogenous) market structure. We also test the prediction of the “firm-colocation” effect, which occurs only in the endogenous market structure: a reduction in Home tariffs encourages not only entry of Foreign firms but also entry of Home firms into the vertically-related markets. We find that the estimation results are consistent with the prediction in the endogenous market structure. In particular, the extensive margin accounts for a larger part of China’s export growth after joining the WTO; and China’s WTO accession encourages entry of Japanese firms as well as entry of Chinese firms. The second prediction is concerned with the impact of market thickness of vertically-related markets on optimal tariffs. Our theory shows that optimal tariffs are higher, the thicker (thinner) the Home market relative to the Foreign market in the exogenous (endogenous) market structure. Using the Herfindahl index as a proxy of market thickness in our regression strategy, we provide evidence that Japan’s tariff rates are higher, the higher the Herfindahl indices and thus the thinner Japanese markets, which also supports the prediction in the endogenous market structure.

A handful of papers have investigated trade policy in the context of vertical oligopolies. In the international vertical oligopoly settings, Ishikawa and Lee (1997), Ishikawa and Spencer (1999), and Chen et al. (2004) analyze strategic interactions between firms in vertically related markets and examine the effect of export subsidies for an imported intermediate input on social welfare.
These papers, however, only consider the exogenous market structure and market thickness is invariant to tariffs. In contrast, the market structures are critical in our model since they affect the response of the extensive and intensive margins to tariff changes and hence affect optimal trade policy. In the context of single-stage oligopoly models, Horstmann and Markusen (1986), Venables (1985) and more recently Etro (2011) and Bagwell and Staiger (2012a, b) all show that the endogenous market structure can alter optimal trade policy obtained from the exogenous market structure. We extend this key insight to a vertical oligopoly model and examine how the endogenous market structure affects optimal trade policy by inducing the firm-colocation effect. The current paper is closest to a companion paper (Ara and Ghosh, 2016). While that paper also studies optimal tariffs in a vertical oligopoly model with both exogenous and endogenous market structures, Home and Foreign firms are assumed to use contractual agreements (rather than spot markets), so as to analyze the impact of firms’ bargaining power on optimal tariffs. Although this approach allows us to explore rich implications resulting from search and matching, one of drawbacks is that bargaining power is not observable and measurable, and thus it is not possible to empirically test the theoretical predictions. The current paper instead exploits the availability of market thickness in data and assesses the relevance between our theory and evidence.

In terms of evidence, our finding that tariff reductions increase the extensive margin of trade is similar to that in the previous study using the China Customs data. For example, Feng et al. (2017) find that China’s export growth after China’s WTO entry is exclusively explained by the fact that reductions in trade policy uncertainty allow Chinese firms to enter the export markets. Their focus is, however, on the impact of China’s WTO accession on entry and exit of Chinese firms, i.e., the extensive margin. We find that the response of the extensive margin to tariffs is significantly greater than the intensive margin after China’s WTO accession. More importantly, we provide evidence on the firm-colocation effect that China’s WTO accession encourages entry of Japanese firms as well as entry of Chinese firms, and unilateral tariff reductions drive the shift in the pattern of entry favoring both liberalizing and liberalized countries. While these findings might not be surprising from those in the existing literature (e.g., Bernard et al. 2007), note that we focus only on tariffs rather than distances. Thus the same result would not necessarily hold for different episodes of trade liberalization in other countries (e.g., Buono and Lalanne, 2012). Concerning the impact of market thickness on tariff setting, to the best of our knowledge, there exists no evidence on the impact of market thickness on tariff setting in vertical specialization. The closest work is Broda et al. (2008) who find that the U.S. tariff rates are significantly higher for goods where the U.S. faces higher foreign export supply elasticities. (See Soderbery (2018) for the extension of their estimation method for optimal tariffs.) In contrast to Broda et al. (2008), we do not analyze the role of China’s export supply elasticities in shaping Japan’s tariff rates. This is not because our welfare-maximizing formula for tariffs is irrelevant to these elasticities, but because our theory sheds light on the effect of market thickness on tariff setting. We find that Japan’s tariff rates are significantly higher for industries where not only is China’s export market thinner, but also Japan’s import market is thinner.
2 Model

Consider two countries, Home and Foreign, with multiple industries, indexed by \( j \in \{1, 2, \ldots, J\} \). The two countries engage in vertical specialization: Home specializes in final goods and Foreign specializes in intermediate inputs. In industry \( j \), there are \( m_j \) identical final-good producers in Home, while there are \( n_j \) identical intermediate-input suppliers in Foreign. Both final goods and intermediate inputs are homogeneous and transacted through markets. Foreign firms produce intermediate inputs with constant marginal cost \( c_j \) and ship to Home with a specific tariff rate \( t_j \). Home firms transform intermediate inputs into final goods with constant marginal cost \( c^d_j \), which is normalized to zero for simplicity. We assume that production of one unit of final goods requires one unit of intermediate inputs. In addition to these production costs, upon entry, Home firms and Foreign firms incur fixed entry costs \( K_{Hj} \) and \( K_{Fj} \) respectively.

There is a unit mass of consumers with a quasi-linear utility function, \( U = \sum_j U_j(Q_j) + q_0 \), where \( Q_j \) is a imperfectly competitive final good produced in industry \( j \) by using an intermediate input and \( q_0 \) is a perfectly competitive numeraire good. If consumers’ income is sufficiently high, maximizing the utility function subject to the budget constraint gives us a demand function for the final good, \( Q_j = Q(P_j) \). We will often work with an inverse demand function, \( P_j = P(Q_j) \). Assume that (i) \( Q(P_j) \) is twice continuously differentiable; and (ii) \( Q'(P_j) < 0 \) for all \( P_j \in (0, \hat{P}_j) \) where \( \hat{P}_j \equiv \lim_{Q_j \to 0} P(Q_j) \) and \( Q(P_j) = 0 \) for \( P_j \geq \hat{P}_j \). Then, we have that (i) \( P_j = P(Q_j) \) is twice continuously differentiable; and (ii) \( P'(Q_j) < 0 \) for all \( Q_j \geq 0 \). For a sharper characterization, we mainly assume that the final good is consumed only in Home and that a Foreign government does not undertake trade policy, but none of the key results relies on these assumptions; see Ara and Ghosh (2016) for these kinds of extensions in the similar setting.

We consider the following three-stage game. In the first stage, a Home government sets a specific tariff rate \( t_j \) to maximize Home welfare which consists of consumer surplus, Home aggregate profits and tariff revenues. In the second stage, upon paying the fixed entry cost \( K_{Fj} \), Foreign firms enter the intermediate-input market and engage in a Cournot competition where profit-maximizing Foreign firms commit to choose the quantity of the intermediate inputs taking rival firms’ quantity as given. In the third stage, upon paying the fixed entry cost \( K_{Hj} \), Home firms enter the final-good market and engage in a Cournot competition where profit-maximizing Home firms commit to choose the quantity of the final goods taking rival firms’ quantity and the input price as given. This input price, denoted by \( r_j \), is determined at the market clearing level which equals the total amount of the intermediate inputs demanded by Home firms to the total amount of the intermediate inputs supplied by Foreign firms.

In order to illustrate important policy implications and empirically testable predictions, we conduct both the “short-run” analysis and the “long-run” analysis in a unified framework. In the short-run analysis in Section 3, we bypass entry considerations in both sectors of production and assume that the numbers of firms are invariant to a tariff rate. Thus, in this section, the market structure is exogenous in that tariff has no impact on the number of firms. In Section 4,
by contrast, we assume that after observing a tariff rate, firms enter the market. Thus, in this section, the market structure is endogenous in that tariff has an impact on the numbers of firms as well as the quantities.

In what follows, we focus on a particular industry and suppress industry subscripts unless necessary in Sections 3 and 4. Similarly, while the empirical setting treats several time periods, the model is static for simplicity and we suppress time subscripts until Section 5.

3 Exogenous Market Structure

This section considers an environment where the entry costs $K_H, K_F$ have been sunk and entry of firms has taken place, and the numbers of these firms $m, n$ are exogenously fixed. We derive the Subgame Perfect Nash Equilibria (SPNE) in pure strategies of the model. Formal proofs for propositions and lemmas are relegated to the Appendix.

3.1 Cournot Competition

We first analyze the third-stage Cournot competition among Home firms operating in the final-good market. Home firm $i$ chooses $q_i$ to maximize $\left( P(q_i + \sum_{d \neq i} q_d) - r \right) q_i$, taking other Home firms’ quantity and the input price $r(< \bar{P})$ as given. If $q_i > 0$ for all $i = 1, 2, ..., m$, the first-order conditions are

$$P\left(q_i + \sum_{d \neq i} q_d\right) - r + P'(q_i + \sum_{d \neq i} q_d) q_i = 0.$$ 

Let $Q \equiv \sum_d q_d$ denote the aggregate output demanded at any given output price $P \in (0, \bar{P})$. The assumption below ensures that the solution to the maximization problem is unique.

**Assumption 1** The demand function $Q(P)$ is logconcave.

The equivalent assumption in terms of the inverse demand function of final goods is:

**Assumption 1’** $P'(Q) + QP''(Q) \leq 0$ for all $Q \geq 0$.

Assumption 1 holds if and only if marginal revenue is steeper than demand. In the trade literature, this assumption is first introduced in Brander and Spencer (1984a, b) who show that when Home imports from a Foreign monopolist with constant marginal cost, a small increase in tariff improves welfare if and only if Assumption 1’ holds.

In our framework, in addition to guaranteeing uniqueness, Assumption 1’ ensures that the optimal tariff is non-negative at least for some $m > 1$ and $n > 1$. A convenient way to state Assumption 1’ is in terms of elasticity of slope, which is defined as $\epsilon(Q) \equiv \frac{QP''(Q)}{P'(Q)}$. Observe that $\epsilon(Q) \geq -1$ if and only if $P'(Q) + QP''(Q) \leq 0$. This condition is sufficient to prove the main results. For analytical simplicity, we focus on a class of demand functions which not only satisfy Assumption 1 but also satisfy the following:
**Assumption 2** \( \epsilon(Q) \equiv \frac{QP''(Q)}{P'(Q)} = \epsilon \) for all \( Q \geq 0 \).

Note, if \( \epsilon \) is constant for all \( Q(\geq 0) \), \( \epsilon \) is greater than \(-1\) and Assumption 1’ or Assumption 1 is satisfied as well. Although this assumption is admittedly restrictive, any well-known inverse demand function (e.g., linear, constant elasticity, semi-log) satisfies Assumption 2.

Now back to the Cournot competition in the final-good market. If \( r \equiv (0, \bar{P}) \), Assumption 1 or 1’ guarantees that there exists a unique symmetric equilibrium \( q_1 = q_2 = ... = q_m \equiv \hat{q}(>0) \) where \( \hat{q} \) is given by the equation below:

\[
\hat{q} = -\frac{P(\hat{Q}) - r}{P'(\hat{Q})},
\]

where \( \hat{Q} = mq \) is uniquely solves the following equation:

\[
mP(\hat{Q}) + \hat{Q}P'(\hat{Q}) = mr. \tag{3.1}
\]

Let \( \pi_H(q, \hat{q}) \equiv [P(q + (m - 1)\hat{q}) - r]q \) denote the post-entry profit of a Home firm that chooses \( q \) as its quantity given all other \( m - 1 \) firms choose \( \hat{q} \). Suppose that \( \pi_H(q, \hat{q}) \) is pseudoconcave in \( q \) at \( q = \hat{q} \). If \( r \equiv (0, \bar{P}) \), we have that \( q_1 = q_2 = ... = q_m \equiv \hat{q}(>0) \) constitutes the third-stage equilibrium. On the other hand, if \( r \in [\bar{P}, \infty) \), each Home firm \( i \) chooses \( q_i = 0 \) in the third-stage equilibrium (see Ghosh and Morita (2007) for details).

Let \( x_u \) denote the amount of intermediate input produced by upstream firm \( u \) and \( X \equiv \sum_u x_u \) denote the aggregate input demanded at any given input price \( r \equiv (0, \bar{P}) \). Since one unit of final goods requires one unit of intermediate inputs, we have that \( X = Q \). Furthermore, since the input price is determined at the market clearing level and the aggregate amount of final goods produced at any given \( r \equiv (0, \bar{P}) \) is \( Q \), it follows from (3.1) that the inverse demand function for intermediate inputs faced by Foreign firms is

\[
r = P(Q) + \frac{QP'(Q)}{m} \equiv g(X). \tag{3.2}
\]

This inverse demand function satisfies the following properties. First, from \( P'(Q) + QP''(Q) \leq 0 \) (by Assumption 1’), we have that

\[
g'(X) = \frac{(m + 1)P'(Q) + QP''(Q)}{m} = \frac{P'(Q)(m + 1 + \epsilon)}{m} < 0. \tag{3.3}
\]

Thus, the inverse demand function for intermediate inputs is downward-sloping. Second, from \( \epsilon(Q) = \frac{QP''(Q)}{P'(Q)} = \epsilon \) for all \( Q \geq 0 \) (by Assumption 2), we also have that

\[
\frac{Xg''(X)}{g'(X)} = \frac{QP''(Q)(m+1+\epsilon)}{m} \frac{m}{P'(Q)(m+1+\epsilon)} = \epsilon. \tag{3.4}
\]
which shows that the elasticity of slope of input demand is the same as that of final-good demand. Further applying $\epsilon \geq -1$ to (3.4), we have that
\[
g'(X) + Xg''(X) \leq 0, \tag{3.5}
\]
for all $X \geq 0$. This condition is a counterpart to Assumption 1’ for the inverse demand function of intermediate inputs.

Now consider the second-stage Cournot competition among Foreign firms in the intermediate-input market. The inverse demand function faced by Foreign firms in the second stage is given by (3.2). Foreign firm $i$ chooses $x_i$ to maximize $\left[ g \left( x_i + \sum_{u \neq i}^n x_u \right) - c - t \right] x_i$ taking other Foreign firms’ quantity as given. If $x_i > 0$ for all $i = 1, 2, ..., n$, the first-order conditions are
\[
g \left( x_i + \sum_{u \neq i}^n x_u \right) - c - t + g' \left( x_i + \sum_{u \neq i}^n x_u \right) x_i = 0.
\]
Given that $\lim_{X \to 0} g(X) = \bar{P}$ from (3.2), condition (3.5) guarantees that there exists a unique symmetric equilibrium $x_1 = x_2 = ... = x_n \equiv \hat{x} (> 0)$ such that
\[
\hat{x} = -\frac{g(\hat{X}) - c - t}{g'(\hat{X})},
\]
where $\hat{X} = n\hat{x}$ uniquely solves the following equation:
\[
n g(\hat{X}) + \hat{X} g'(\hat{X}) = n(c + t). \tag{3.6}
\]
Let $\pi_F(x, \hat{x}) \equiv [g(x + (n - 1)\hat{x}) - c - t]x$ denote the post-entry profit of an Foreign firm that chooses $x$ as its quantity given all other $n - 1$ firms choose $\hat{x}$. Since $\pi_F(x, \hat{x})$ is strictly concave in $x$ for all $x > 0$ (by virtue of (3.5)) and $r \in (0, \bar{P})$, we have that $x_1 = x_2 = ... = x_n \equiv \hat{x} (> 0)$ constitutes the second-stage equilibrium.

To summarize, in the Cournot competition with given $m$, $n$ and $t$, we have an output vector $(\hat{q}, \hat{Q}, \hat{x}, \hat{X})$ and a price vector $(\hat{P}, \hat{r})$ where
\begin{itemize}
  \item $\hat{Q}$ solves (3.1);
  \item $\hat{X}$ solves (3.6);
  \item $\hat{Q} = \hat{X}$;
  \item $\hat{q} = \frac{Q}{m}, \hat{x} = \frac{X}{n}$;
  \item $\hat{P} \equiv P(\hat{Q})$, $\hat{r} \equiv g(\hat{X})$.
\end{itemize}
The following lemma records comparative statics results with respect to $n$ and $t$. While our focus is on $n$ below, comparative static results with respect to $m$ are similarly obtained (see Appendix).
Lemma 3.1

(i) For a given tariff rate $t$, the aggregate output $\hat{Q}$ and aggregate input $\hat{X}$ are increasing in $n$; while the final-good price $\hat{P}$ and input price $\hat{r}$ are decreasing in $n$; i.e., $\partial \hat{Q} / \partial n = \partial \hat{X} / \partial n > 0$, $\partial \hat{P} / \partial n < 0$, $\partial \hat{r} / \partial n < 0$.

(ii) For a given number of Foreign firms $n$, the aggregate output $\hat{Q}$ and aggregate input $\hat{X}$ are decreasing in $t$; while the final-good price $\hat{P}$ and input price $\hat{r}$ are increasing in $t$; i.e., $\partial \hat{Q} / \partial t = \partial \hat{X} / \partial t < 0$, $\partial \hat{P} / \partial t > 0$, $\partial \hat{r} / \partial t > 0$.

(iii) Let $r^* \equiv \hat{r} - t$ denote the price received by a Foreign firm in equilibrium (for each unit of the intermediate input). Then,

$$\frac{\partial r^*}{\partial t} \leq 0 \iff \frac{\partial \hat{r}}{\partial t} \leq 1 \iff 1 + \epsilon \geq 0.$$

Not surprisingly, $\hat{Q}, \hat{X}$ decrease as $t$ increases. Due to the exogenous market structure where the number of firms $m, n$ is fixed, an increase in $t$ decreases aggregate outputs $\hat{Q}, \hat{X}$ only through the reduction in average outputs per firm $\hat{q}, \hat{x}$. It follows immediately from $\hat{Q} = mq$ and $\hat{X} = nx$ that $\hat{q} = \frac{\hat{Q}}{m} = \frac{m \hat{q}}{m}$ and $\hat{x} = \frac{\hat{X}}{n} = \frac{n \hat{x}}{n}$, which in turn implies from Lemma 3.1(ii) that

$$\frac{\partial \hat{q}}{\partial t} < 0, \quad \frac{\partial \hat{x}}{\partial t} < 0. \quad (3.7)$$

The next section will show that entry considerations do not necessarily lead to this seemingly unsurprising result in the endogenous market structure.

Since $\hat{X}$ decreases as $t$ increases, $\hat{r}$ increases as $t$ increases. Note, however, that $\frac{\partial \hat{r}}{\partial t} - 1 \leq 0$ or equivalently $\frac{\partial r^*}{\partial t} \leq 0$ as long as the demand is logconcave. For all such demand functions, the pass-through of tariff to an intermediate-input price faced by Home producers is less than complete. Foreign firms absorb part of the tariff increase which acts like a terms-of-trade gain for Home. While $r^*$ is an input price internal to each firm, a reduction in $r^*$ hurts Foreign firms and benefits Home firms. Hence, we refer to a decrease in $r^*$ as a terms-of-trade improvement in the paper, though we are aware that $r^*$ is more like firms’ terms-of-trade (rather than countries’ terms-of-trade). The terms-of-trade improvement creates a rationale for Home to set a positive tariff.

It is worth mentioning that there is a “double marginalization” effect at work in our model of vertical specialization, like Ishikawa and Lee (1997) and Ishikawa and Spencer (1999). Imperfect competition in the final-good market creates a wedge between the price of final goods and its marginal cost $\hat{P} - \hat{r}$, while imperfect competition in the intermediate-input market creates a wedge between the price of intermediate inputs and its marginal cost $\hat{r} - c - t$. The inefficiency associated with this double marginalization effect plays a key role in characterizing the optimal tariff set by the Home government.
3.2 Tariffs

In the first stage, the Home government chooses a tariff rate $t$ to maximize Home welfare $W_H$, anticipating the output vector $(\hat{q}, \hat{Q}, \hat{x}, \hat{X})$ and the price vector $(\hat{P}, \hat{r})$ determined in the Cournot competition. In the SPNE of the first-stage subgame, Home welfare is given by

$$W_H = \sum_j \left[ \left( \int_0^{\hat{Q}_j} P(y)dy - P(\hat{Q}_j)\hat{Q}_j \right) + \left( P(\hat{Q}_j) - \hat{r}_j \right) \hat{Q}_j + t_j \hat{X}_j \right],$$

where industry subscript $j$ is re-attached. The first, second, and third terms represent consumer surplus, Home profits, and tariff revenue, respectively. Using $r_j^* = \hat{r}_j - t_j$ defined in Lemma 3.1 and using $\hat{Q}_j = \hat{X}_j$, the above expression is simplified as follows:

$$W_H = \sum_j \left[ \int_0^{\hat{Q}_j} P(y)dy - r^*_j \hat{X}_j \right].$$

Differentiating this welfare expression with respect to $t_j$ and using $\partial \hat{Q}_j / \partial t_j = \partial X_j / \partial t_j$, we get

$$\frac{dW_H}{dt_j} = (P(\hat{Q}_j) - r^*_j) \frac{\partial \hat{Q}_j}{\partial t_j} - \frac{\partial r^*_j}{\partial t_j} \hat{X}_j.$$

The first term captures the welfare loss due to the tariff-induced output reduction ($\frac{\partial \hat{Q}_j}{\partial t_j} < 0$). Home consumers value the final goods at $P(\hat{Q}_j)$ while effectively it costs $r^*_j (P(\hat{Q}_j))$ to produce (from Home’s perspective). This price-cost margin $P(\hat{Q}_j) - r^*_j$ multiplied by the amount of output lost $\frac{\partial \hat{Q}_j}{\partial t_j}$ is the magnitude of welfare loss. The second term captures the welfare gains from the terms-of-trade improvement ($\frac{\partial r^*_j}{\partial t_j} < 0$). While the optimal tariff strikes a balance between these two effects as in the previous literature, the numbers of firms $m_j, n_j$ play a key role in delineating the relative importance of the two effects in our vertical oligopoly model.

Applying the comparative statics in Lemma 3.1, $W_H$ is strictly concave in $t_j$ and the optimal tariff is uniquely welfare-maximizing. Setting $\frac{dW_H}{dt_j} = 0$ and solving for $t_j$ gives the expression for the optimal tariff which is presented later in Proposition 3.1. Here we first focus on the sign of the optimal tariff. Using $\frac{\partial r_j^*}{\partial t_j} = \frac{\partial \hat{r}_j}{\partial t_j} - 1$ and $\frac{\partial \hat{Q}_j}{\partial t_j} = m_j \frac{\partial \hat{q}_j}{\partial t_j}$, we can express $\frac{dW_H}{dt_j}$ as follows:

$$\frac{dW_H}{dt_j} = (P(\hat{Q}_j) - \hat{r}_j)m_j \frac{\partial \hat{q}_j}{\partial t_j} + \left( 1 - \frac{\partial \hat{r}_j}{\partial t_j} \right) \hat{X}_j + t_j \frac{\partial \hat{X}_j}{\partial t_j}. \quad (3.8)$$

Noting $\frac{\partial \hat{X}_j}{\partial t_j} < 0$ from Lemma 3.1 in (3.8), it follows immediately that the optimal tariff is strictly positive (negative) if and only if

$$(P(\hat{Q}_j) - \hat{r}_j)m_j \frac{\partial \hat{q}_j}{\partial t_j} + \left( 1 - \frac{\partial \hat{r}_j}{\partial t_j} \right) \hat{X}_j > ( <) 0. \quad (3.9)$$
Equation (3.8) indicates that the numbers of firms play a key role in determining the sign of the optimal tariff. Suppose that for a given $m_j$, the number of Foreign firms $n_j$ is arbitrarily large so that the Foreign market is perfectly competitive. Then, the input price equals its marginal cost ($\hat{r}_j = c_j + t_j$) and, as a result, $\left(1 - \frac{\partial r_j}{\partial t_j}\right) \hat{X}_j = -\frac{\partial r_j}{\partial t_j} \hat{X}_j = 0$, i.e., the terms-of-trade motive vanishes. Only the harmful effect of the tariff remains. An import subsidy raises Home welfare by raising output and indeed the optimal tariff is negative. More generally, when $n_j$ is arbitrarily large, Home captures all profits in Cournot competition of the final-good market. The situation is like a domestic, single-stage, Cournot oligopoly with $m_j$ firms. A positive subsidy increases welfare in an oligopoly setup by narrowing the wedge between price and marginal cost.

Conversely, suppose that for a given $n_j$, the number of Home firms $m_j$ is arbitrarily large so that the Home market is perfectly competitive. Then, the final-good price is equal to its marginal cost ($P(\hat{Q}_j) = \hat{r}_j$) and, as a result, $(P(\hat{Q}_j) - \hat{r}_j) m_j \frac{\partial q_j}{\partial t_j} = 0$, i.e., the welfare loss due to the tariff-induced output reduction vanishes. This is equivalent for Home to importing the final good from Foreign and its welfare is composed of consumer surplus and tariff revenues. In such a case, the reduction in profits is borne only by Foreign producers and the sign of the optimal tariff is determined exclusively by the terms-of-trade motive, or by the sign of $1 - \frac{\partial r_j}{\partial t_j} = -\frac{\partial r_j}{\partial t_j}$. As the pass-through from tariff to domestic prices is incomplete for all logconcave demand functions, $1 - \frac{\partial r_j}{\partial t_j} > 0$ holds, which implies that the optimal tariff is strictly positive.

The above intuition means that the Home optimal tariff is positive (negative) if the number of Foreign firms $n_j$ is relatively smaller (larger) than the number of Home firms $m_j$. (Note that the channel is not operative for differentiated goods with CES demand systems, as the price-cost margins are constant.) This comes out cleanly in terms of the price-cost margin ratio $\frac{\hat{P}_j - \hat{r}_j}{\hat{r}_j - c_j - t_j}$. Using (3.2) and (3.6), this ratio is expressed as

$$\frac{\hat{P}_j - \hat{r}_j}{\hat{r}_j - c_j - t_j} = \frac{\hat{Q}_j P'(\hat{Q}_j)}{m_j} = \frac{n_j}{m_j + 1 + \epsilon_j}.$$  

(3.10)

Clearly, if this ratio is relatively lower (higher), the terms-of-trade improvement effect in (3.9) is more (less) likely to dominate the output reduction effect and the sign of the optimal tariff is positive (negative). Further, since this ratio is strictly increasing in $n_j$, there is a range of values such that the optimal tariff is strictly decreasing in $n_j$. Analyzing (3.8) further gives us a more precise characterization of the optimal tariff.

**Proposition 3.1**

Let $i_j$ denote the optimal tariff in industry $j$. At $t_j = \hat{i}_j$ the following holds:

$$\hat{i}_j = -\hat{Q}_j P'(\hat{Q}_j) \left(\frac{(1 + \epsilon_j)(m_j + 1 + \epsilon_j) - n_j}{m_j n_j}\right),$$  

(3.11)

where $\hat{Q}_j$ is the aggregate output evaluated at $t_j = \hat{i}_j$. Furthermore,
(i) There exists $n_j^*$ such that

$$\hat{t}_j \leq 0 \iff n_j \leq n_j^* \equiv (1 + \epsilon_j)(m_j + 1 + \epsilon_j).$$

(ii) $\hat{t}_j$ is monotonically decreasing in $n_j$ and increasing in $m_j$.

As in the previous work of horizontal specialization, the Home optimal tariff is characterized by the domestic distortion effect and the terms-of-trade effect in vertical specialization (see (3.9)). This means that if the Foreign export supply elasticity is infinite and Home has no market power, $\frac{\partial r_j^*}{\partial t_j} = 0$ and the optimal tariff is low. As the Foreign export supply elasticity is smaller and Home has more market power, the terms-of-trade effect is greater and the optimal tariff is higher (see, e.g., Broda et al., 2008). We can interpret this well-known result in terms of market thickness rather than the export supply elasticities. If the Foreign market is arbitrarily thick, $\frac{\partial r_j^*}{\partial t_j} = 0$ and the optimal tariff is low. As the Foreign market is thinner, the terms-of-trade effect is greater and the Home optimal tariff is higher. The effect is similar between Foreign export supply elasticities and Foreign market thickness in our vertical oligopoly model.

This line of reasoning also allows us to investigate the impact of Home market thickness on the Home optimal tariff. If the Home market is arbitrarily thick, $P(\hat{Q}_j) = \hat{r}_j$ and the optimal tariff is high since only the terms-of-trade effect remains. As the Home market is thinner, the domestic distortion effect is greater and the Home optimal tariff is lower. Thus, monopoly power in the domestic market eliminates a motive for the use of import tariffs.

In Section 5, we test this prediction. Suppose that market thickness varies across industries. If the numbers of firms $m_j, n_j$ are invariant to tariff changes and only the quantities $q_j, x_j$ are responsible to trade policy, the optimal tariff $\hat{t}_j$ is higher in industry $j$ where the Foreign market is thinner or the Home market is thicker. While the role of the Foreign market has been analyzed in the existing literature (Broda et al., 2008; Soderbery, 2018), the role of the Home market in vertical specialization has been less explored, which is shown to depend on the market structure. Further, in contrast to the previous studies, we explore how the impact of tariffs on the extensive and intensive margins influences the Home optimal tariff.

As an illustrative example, consider the following class of inverse demand functions: $P(Q_j) = a_j - Q_j^{by}$. Note that $b_j = 1$ for linear demand and $b_j > (<)1$ for strictly concave (convex) demand. The elasticity of slope of demand is constant and denoted by $\epsilon_j = b_j - 1$. In this specific case, the optimal tariff in (3.11) is expressed as

$$\hat{t}_j = \left[ \frac{b_j(m_j + b_j) - n_j}{m_jn_j + b_j(b_j + 1)(m_j + b_j)} \right] (a_j - c_j)b_j.$$

Observe that the property of the optimal tariff in Proposition 3.1 holds for this specific class of inverse demand functions. In particular, (i) $\hat{t}_j$ is positive (negative) if $n_j > (<)b_j(m_j + b_j)$; and (ii) $\hat{t}_j$ is strictly increasing (decreasing) in $m_j$ ($n_j$).
3.3 Profits

Aggregate profits earned by Home firms and Foreign firms respectively are given by

\[ \hat{\Pi}_H \equiv m\hat{\pi}_H = [P(\hat{Q}) - \hat{r}]\hat{Q}, \quad \hat{\Pi}_F \equiv n\hat{\pi}_F = [r(\hat{X}) - c - \hat{t}]\hat{X}, \]

where \( \hat{\pi}_H \equiv (P(\hat{Q}) - \hat{r})\hat{q} \) and \( \hat{\pi}_F \equiv (\hat{r} - c - \hat{t})\hat{x} \) are the post-entry profit of each Home firm and Foreign firm. Here we are interested in the impact of an increase in the number of Foreign firms \( n \) on these aggregate profits \( \hat{\Pi}_H, \hat{\Pi}_F \). Differentiating \( \hat{\Pi}_H \) with respect to \( n \) yields

\[
\frac{d\hat{\Pi}_H}{dn} = -(2 + \epsilon)\hat{q}P'(\hat{Q})\frac{d\hat{Q}}{dn} > 0,
\]

where \( \frac{d\hat{Q}}{dn} = \frac{\partial \hat{Q}}{\partial n} + \frac{\partial \hat{Q}}{\partial \hat{t}}\frac{d\hat{t}}{dn} > 0 \). Thus, an increase in \( n \) raises Home profits, because it lowers the imported input price \( \hat{r} \). On the other hand, differentiating \( \hat{\Pi}_F \) with respect to \( n \) yields

\[
\frac{d\hat{\Pi}_F}{dn} = (n - 1)\hat{x}g'(\hat{X})\frac{d\hat{X}}{dn} - \frac{d\hat{t}}{dn}\hat{X}, \tag{3.12}
\]

where \( \frac{d\hat{X}}{dn} = \frac{\partial \hat{X}}{\partial n} + \frac{\partial \hat{X}}{\partial \hat{t}}\frac{d\hat{t}}{dn} > 0 \). An increase in \( n \) has two opposing effects on \( \hat{\Pi}_F \). First, an increase in \( n \) amplifies competition in the intermediate-input market and lowers Foreign profits (competition effect), which exists even when the tariff is exogenously set. Second, an increase in \( n \) lowers the optimal tariff \( \hat{t} \) and raises Foreign profits (tariff-reduction effect). Surprisingly, we find that for arbitrarily large \( m \), the latter effect can dominate the former effect.

**Proposition 3.2**

An increase in the number of Foreign firms might lead to the higher Foreign aggregate profits. For arbitrarily large \( m \), \( \left. \frac{d\hat{\Pi}_F}{dn} \right|_{m=\infty} > 0 \) if

\[
n < \frac{1 + \sqrt{1 + 4(1 + \epsilon)(2 + \epsilon)}}{2}.
\]

Proposition 3.2 shows that an indirect increase in the Foreign aggregate profits due to a lower optimal tariff might outweigh a direct decrease in these Foreign profits due to more competition in the Foreign market. This situation is more likely when the number of the Foreign aggregate firms \( n \) is smaller or the curvature of the inverse demand \( \epsilon \) is bigger. Note, however, that this claim applies only for the Foreign aggregate profits \( \hat{\Pi}_F \), and for the individual Foreign profits \( \hat{\pi}_F \), the direct decrease always outweighs the indirect increase.

To better appreciate this proposition, consider \( P(Q) = a - Q^b \) for which \( \epsilon = b - 1 \). For linear demand \( (b = 1) \), \( \lim_{m \to \infty} \frac{d\hat{\Pi}_F}{dn} \geq 0 \) if and only if \( n \leq 2 \), which implies that for arbitrary large \( m \), the Foreign aggregate profits increase as the number of Foreign firms increases from one to two. As demand functions is more concave, this counter-intuitive outcome is more likely.
4  Endogenous Market Structure

In Section 3, we have assumed that the numbers of Home and Foreign firms are fixed. Since \( m \) and \( n \) are fixed, these numbers do not vary with tariff rates. Now we consider an environment where \( m \) and \( n \) are endogenously determined and tariffs are set prior to entry decisions. Here, in addition to the the direct effect on quantities and prices, tariffs also indirectly affect quantities and prices by influencing the market structure. In particular, in vertical specialization where Home output and Foreign input are complements, Home tariffs lead to not only the reduction in the number of Foreign firms but also the reduction in the number of Home firms. This “firm-colocation” effect plays a key role in the endogenous market structure.

In the context of single-stage oligopoly models, Horstmann and Markusen (1986), Venables (1985) and more recently Etro (2011) and Bagwell and Staiger (2012a, b) all have shown that the endogenous market structure can drastically alter the optimal trade policy obtained from the exogenous market structure. Like these preceding papers, we also find that free entry can affect the relationship between the optimal tariff and market thickness. In our vertical oligopoly model, tariffs do not always lead to the terms-of-trade improvement or the average output reduction in the endogenous market structure, which stands in contrast to the exogenous market structure. Due to these endogenous differences caused by free entry, we show that there is a non-monotone relationship between the optimal tariff and market thickness. The nature of non-monotonicity depends particularly on the curvature of demand function.

The timing of events is outlined in Section 2. In contrast to Section 3 where entry decisions are ignored by assumption of the short run, this section explicitly takes account of entry decisions so that the numbers of entrants in each market are variant to a tariff rate. As before, we derive the Subgame Perfect Nash Equilibria (SPNE) in pure strategies of the model and focus on a class of demand functions which satisfy Assumptions 1 and 2.

4.1 Cournot Competition

Let us start with analyzing the third stage. The Cournot competition in the final-good market works exactly the same way as before and the unique symmetric equilibrium in this stage is characterized by \( q_1 = q_2 = \ldots = q_m = \hat{q} \) such that

\[
\hat{q} = -\frac{P(\hat{Q}) - r}{P'(\hat{Q})},
\]

where \( \hat{Q} \) satisfies the following for any given \( m \):

\[
mP(\hat{Q}) + \hat{Q}P'(\hat{Q}) = mr. \tag{4.1}
\]

In addition to (4.1), the number of Home firms \( m \) is endogenously determined as there is free entry of firms. Recall from Section 2 that the entry cost of a Home firm is \( K_H \). In the long run
where entry is unrestricted, entry occurs until the post-entry profit of Home firms equals the entry cost. Let $\pi_H(m) \equiv (P(mq) - r)q$ denote the post-entry profit of Home firms in the SPNE of the third stage. Then the free entry condition in the final-good market is given by $\pi_H(\hat{m}) = K_H$:

$$[P(\hat{m}q) - r]q = K_H.$$ 

Aggregating this condition for all $\hat{m}$ Home firms, $\hat{m}$ satisfies the following for any given $Q$:

$$[P(Q) - r]Q = \hat{m}K_H.$$  (4.2)

We assume that $K_H \leq \pi_H(1) \equiv \bar{K}_H$, which guarantees that at least one Home firm enters in the equilibrium.

**Assumption 3**  $K_H \leq \bar{K}_H$.

Since $\pi_H(m)$ is continuous in $m$ and strictly decreasing in $m$ for all $m > 1$, Assumption 3 also ensures that $\hat{m}$ uniquely exists in the SPNE of the third-stage subgame.

Now let us turn to analyzing the second stage. Given the SPNE of the third-stage subgame above, the inverse demand function for intermediate inputs $X$ faced by Foreign firms (3.2) holds in both the short-run and long-run equilibria. Thus,

$$r = P(Q) + \frac{QP'(Q)}{m} \equiv g(X, m).$$

While we defined $r \equiv g(X)$ in (3.2) since the number of firms is exogenous in the short run, we explicitly define $r$ as a function of $m$ as well as $X$ since $m$ is endogenous in the long run. This inverse demand function satisfies

$$g_x(X, m) \equiv \frac{\partial g(X, m)}{\partial X} = \frac{(m + 1 + \epsilon)P'(Q)}{m} < 0,$$

$$g_m(X, m) \equiv \frac{\partial g(X, m)}{\partial m} = -\frac{QP'(Q)}{m^2} > 0,$$

$$g_{xm}(X, m) \equiv \frac{\partial^2 g(X, m)}{\partial X \partial m} = -\frac{(1 + \epsilon)P'(Q)}{m^2} > 0.$$ 

While the first is the same as (3.3), the second shows that an increase in the number of Home firms $m$ shifts the inverse demand upwards.

In the second stage, the Cournot competition in the intermediate-input market works almost the same way as before (except for the expression of input price $r = g(X, m)$) and the unique symmetric equilibrium in this stage is characterized by $x_1 = x_2 = \ldots = x_n \equiv \hat{x}$ such that

$$\hat{x} = -\frac{g(\hat{X}, m) - c - t}{g_x(\hat{X}, m)},$$

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where $\hat{X}$ satisfies the following for any given $n$:

$$ng(\hat{X}, m) + \hat{X}g_x(\hat{X}, m) = n(c + t). \quad (4.3)$$

In addition to (4.3), the number of Foreign firms $n$ is also endogenously determined by equaling the number of Foreign firms to the entrance cost. Let $\pi_F(n) \equiv (g(nx) - c - t)x$ denote the post-entry profit of Foreign firms in the SPNE of the second stage. Then the free entry condition in the intermediate-input market is given by $\pi_F(\hat{n}) = K_F$:

$$[g(\hat{nx}, m) - c - t]x = K_F.$$ 

Aggregating this condition for all $\hat{n}$ Foreign firms, $\hat{n}$ satisfies the following for any given $X$:

$$[g(X, m) - c - t]x = \hat{n}K_F. \quad (4.4)$$

We assume that $K_F \leq \pi_F(1) \equiv \bar{K}_F$, which guarantees that at least one Foreign firm enters in the equilibrium.

**Assumption 4** $K_F \leq \bar{K}_F$.

As in the case of $\hat{m}$, Assumption 4 ensures that $\hat{n}$ uniquely exists in the SPNE of the second-stage subgame. Further, following Section 3, we can show that $q_1 = q_2 = \ldots = q_m = \hat{q}(> 0)$ constitutes the third-stage equilibrium whereas $x_1 = x_2 = \ldots = x_n = \hat{x}(> 0)$ constitutes the second-stage equilibrium.

To summarize, in the Cournot competition with given $K_H$, $K_F$ and $t$, we have an output vector $(\hat{q}, \hat{Q}, \hat{x}, \hat{X})$, a price vector $(\hat{P}, \hat{r})$ and a number vector $(\hat{m}, \hat{n})$ where

- $\hat{Q}$ solves (4.1);
- $\hat{X}$ solves (4.3);
- $\hat{m}$ solves (4.2);
- $\hat{n}$ solves (4.4);
- $\hat{Q} = \hat{X}$;
- $\hat{q} = \frac{\hat{Q}}{\hat{m}}, \hat{x} = \frac{\hat{X}}{\hat{n}}$;
- $\hat{r} \equiv g(\hat{X}, \hat{m}), \hat{P} \equiv P(\hat{Q})$.

Note that (4.1) and (4.3) are the first-order conditions that hold even in the short run, whereas (4.2) and (4.4) are the free-entry conditions that hold only in the long run. These two conditions jointly pin down the numbers of firms as well as the quantities produced by these firms in the long-run equilibrium.
Figure 4.1 illustrates the equilibrium outcomes which can be solved from the first-order and free-entry conditions. An equilibrium in the SPNE of the third-stage subgame is a vector \((\hat{Q}, \hat{m})\), which solves (4.1) and (4.2) in the final-good market in Home. In the second quadrant of Figure 4.1, \(A^D D^D\) depicts (4.1) and \(B^D D^D\) depicts (4.2) in \((m, Q)\) space. The fact that \(B^D D^D\) is steeper than \(A^D D^D\) follows from noting that
\[
\frac{dQ}{dm}_{A^D D^D} = \frac{q}{m + 1 + \epsilon} < \frac{dQ}{dm}_{B^D B^D} = 2q \frac{m}{2 + \epsilon}.
\]
Point \(E^D\), the intersection of \(A^D D^D\) and \(B^D B^D\), uniquely determines the equilibrium vector \((\hat{Q}, \hat{m})\). From this, the number of Home firms and their average outputs are given by
\[
\hat{m} = \sqrt{-\frac{P'(\hat{Q})\hat{Q}^2}{K_H}}, \quad \hat{q} = \sqrt{-\frac{K_H}{P'(\hat{Q})}}.
\]

Similarly, an equilibrium in the SPNE of the second-stage subgame is a vector \((\hat{X}, \hat{n})\), which solves (4.3) and (4.4) in the intermediate-input market in Foreign. In the first quadrant of Figure 4.1, \(A^U U^U\) depicts (4.3) and \(B^U B^U\) depicts (4.4) in \((n, X)\) space. The fact that \(B^U B^U\) is steeper than \(A^U U^U\) follows from noting that
\[
\frac{dX}{dn}_{A^U U^U} = \frac{x}{n + 1 + \epsilon} < \frac{dX}{dn}_{B^U B^U} = 2x \frac{\hat{m}}{2 + \epsilon}.
\]
Point \(E^U\), the intersection of \(A^U U^U\) and \(B^U B^U\), uniquely determines the equilibrium vector \((\hat{X}, \hat{n})\). From this, the number of Foreign firms and their average outputs are given by
\[
\hat{n} = \sqrt{-\frac{g_x(\hat{X}, \hat{m})\hat{X}^2}{K_F}}, \quad \hat{x} = \sqrt{-\frac{K_F}{g_x(\hat{X}, \hat{m})}}.
\]
To characterize the equilibrium, we often find it useful to work with the first-order condition and the free-entry condition that are expressed in the relative terms. Dividing (4.1) by (4.3) gives the first-order condition in the relative terms of Home:

$$\frac{P(Q) - r}{g(X, m) - c - t} = \left(\frac{P'(Q)}{g_x(X, m)}\right) z,$$  \hspace{1cm} (4.5)

where $z \equiv \frac{n}{m}$ is the relative market thickness. Note that (4.5) holds even in the short run. Further, dividing (4.2) by (4.4) gives the free-entry condition in the relative terms of Home:

$$\frac{P(Q) - r}{g(X, m) - c - t} = \frac{k}{z},$$  \hspace{1cm} (4.6)

where $k \equiv \frac{K_H}{K_F}$ is the relative entry cost. Since these entry costs have been sunk in the short run, (4.6) holds only in the long run.

Figure 4.2 illustrates the equilibrium outcome which can be solved from (4.5) and (4.6). In the figure, $AA$ depicts (4.5) and $BB$ depicts (4.6) in $(z, \frac{P-Q}{r-c-t})$ space. Clearly, $AA$ is upward-sloping and $BB$ is downward-sloping. Point $E$, the intersection of $AA$ and $BB$, uniquely determines the equilibrium vector $(\hat{z}, \frac{\hat{P}-r}{\hat{r}-c-t})$. Noting that the system of equations (4.5) and (4.6) can be solved for this vector, we get

$$\hat{z} = \sqrt{k g_x(\hat{X}, \hat{m}) P'(\hat{Q})}, \quad \frac{\hat{P} - \hat{r}}{\hat{r} - c - t} = \sqrt{k g_x(\hat{X}, \hat{m})}.$$  \hspace{1cm} (4.7)

Given the equilibrium outcome, we next examine comparative statics with respect to $K_H$ and $t$. While we focus on $K_H$ below, it is possible to get comparative statics results with respect to $K_F$ (see Appendix).

1From (3.3), $\frac{P'(Q)}{g_x(X, m)} = \frac{m}{m+1+r}$, and (4.5) is also expressed as $\frac{P(Q) - r}{g(X, m) - c - t} = \frac{n}{m+1+r}$, which is the same as (3.10).
Effect of a change in entry cost: Recall in the short run that we examine comparative statics with respect to the number of Foreign firms $n$ (in addition to $t$). In the long run, however, since the numbers of firms are endogenously determined, we cannot conduct these comparative statics. A natural candidate of the exogenous variable that shapes the numbers of firms $\hat{m}, \hat{n}$ (and the relative market thickness $\hat{z}$) would be the fixed entry cost. We interpret this fixed entry cost as competition policy broadly defined, or policies in general – as well as other institutional features of an economy – that make it difficult to start a business. The comparative statics with respect to $K_H$ then allow us to demonstrate that the market thickness is endogenously constrained by the limits given by $K_H$.

Let us consider the impact on $K_H$ on the equilibrium vector by noting the system of equations (4.1) – (4.4). Since $K_H$ only appears in (4.2), it follows that, as $K_H$ increases, $m$ must decrease for any given $Q$ and (4.2) shifts to the right in the second quadrant of Figure 4.1. Consequently, we find that $m$ must decrease for any given $Q$ and both $\hat{Q}$ and $\hat{m}$ decline in the new equilibrium. Further, since $Q = X$, a decline in $Q$ implies a decline in $X$, which successively induces changes in (4.3) and (4.4). It is easily shown that, as $X$ decreases, both (4.3) and (4.4) shift to the left in the first quadrant of Figure 4.1. As a result, both $\hat{X}$ and $\hat{n}$ decline. Note importantly that when it is difficult to start a business in Home, this discourages not only entry of Home firms but also entry of Foreign firms in vertical specialization.

This impact of $K_H$ can be also found by noting the system of equations (4.5) – (4.6). Observe that $k = K_H/K_F$ only appears in (4.6). Then it follows that, as $K_H$ increases, only (4.6) shifts up while keeping (4.5) unchanged. As a result, both $\hat{z}$ and $\frac{\hat{P} \cdot \hat{r}}{c - t}$ increase in the new equilibrium. The fact that $\hat{z}$ increases with $K_H$ implies that $\hat{m}$ declines relatively more than $\hat{n}$, because Home (Foreign) firms are directly (indirectly) affected by an increase in $K_H$.

The following lemma summarizes comparative statics results with respect to $K_H$.

**Lemma 4.1**

(i) For a given tariff rate $t$, the aggregate output $\hat{Q}$ and aggregate input $\hat{X}$ are decreasing in $K_H$; while the final-good price $\hat{P}$ and input price $\hat{r}$ are increasing in $K_H$; i.e., $\partial \hat{Q}/\partial K_H < 0$, $\partial \hat{P}/\partial K_H > 0$, $\partial \hat{r}/\partial K_H > 0$.

(ii) For a given tariff rate $t$, the number of firms $\hat{m}$, $\hat{n}$ is decreasing in $K_H$ and the market thickness $\hat{z} \equiv \hat{n}/\hat{m}$ is increasing in $K_H$; i.e., $\partial \hat{m}/\partial K_H < 0$, $\partial \hat{n}/\partial K_H < 0$ and $\partial \hat{z}/\partial K_H > 0$.

Effect of a change in tariff rate: Next we consider the effect of a change in tariff rate $t$. Observe that $t$ only appears in (4.3) and (4.4). Then, as $t$ increases, $n$ must decrease for any given $X$ and both (4.3) and (4.4) curves shift to the left in the first quadrant of Figure 4.1. Consequently, $n$ must decrease for any given $X$ and both $\hat{X}$ and $\hat{n}$ decline. Further, since $Q = X$, a decline in $X$ implies a decline in $Q$, which successively induces changes in (4.1) and (4.2). We
find that, as \( Q \) decreases, \( m \) must decrease and both (4.1) and (4.2) shift to the right in the second quadrant of Figure 4.1. As a result, both \( \hat{Q} \) and \( \hat{m} \) decline.

Recall from Section 3, a tariff lowers equilibrium outputs \( \hat{Q}, \hat{X} \) and raises equilibrium prices \( \hat{P}, \hat{\rho} \) even when the numbers of Home and Foreign firms are exogenously given. Here, a tariff also discourages entry in both sectors of production and lowers the numbers of Home and Foreign firms \( \hat{m}, \hat{n} \). This effect on entry lowers outputs and raises prices even further.

It is worth emphasizing that, in vertical specialization where Home firms’ output and Foreign firms’ input are complements, trade policy gives rise to the similar impact on entry of Home firms and Foreign firms through the “firm-colocation” effect: if a tariff on Foreign intermediate inputs discourages entry of Foreign firms, it also discourages entry of Home firms \((\frac{\partial \hat{m}}{\partial t} < 0, \frac{\partial \hat{n}}{\partial t} < 0)\). This does not occur in horizontal specialization where Home firms’ output and Foreign firms’ output are substitutes, in which case it is well-known that trade policy gives rise to the opposite impact on entry of Home firms and Foreign firms through the “firm-delocation” effect (see Horstmann and Markusen, 1986; Venables, 1985; and the synthesis in Helpman and Krugman, 1989, Ch. 7): if a tariff on Foreign final goods discourages entry of Foreign firms, it encourages entry of Home firms \((\frac{\partial \hat{m}}{\partial t} > 0, \frac{\partial \hat{n}}{\partial t} < 0)\). This channel is recently re-examined by Bagwell and Staiger (2012a, b) to study the long-run effect of trade agreements on entry of Home firms and Foreign firms in the linear Cournot delocation model. However, to the best of our knowledge, the Cournot colocation model has received relatively less attention in the trade policy literature, despite a growing body of empirical evidence that documents the importance of vertical specialization.

The impact of \( t \) can be also found in the relative terms depicted in Figure 4.2. As \( t \) increases, both (4.5) and (4.6) shift down, and thus \( \hat{z} \) increases but \( \frac{\hat{P} - \hat{\rho}}{\hat{r} - c - t} \) decreases in the new equilibrium. The fact that \( \hat{z} \) increases with \( t \) implies that \( \hat{m} \) declines relatively more than \( \hat{n} \). Although this result would depend on constant elasticity of slope (see (4.7)), note that it occurs only in vertical specialization. If we instead consider horizontal specialization, a tariff has the opposing impact on \( \hat{m} \) and \( \hat{n} \) as seen above, and as a result, \( \hat{z} = \frac{n}{m} \) necessarily decreases with \( t \).

The following lemma records some important comparative statics results with respect to \( t \).

**Lemma 4.2**

(i) For a given entry cost \( K_H \), the aggregate output \( \hat{Q} \) and aggregate input \( \hat{X} \) are decreasing in \( t \), while the final-good price \( \hat{P} \) and input-price \( \hat{\rho} \) are increasing in \( t \); i.e., \( \partial \hat{Q}/\partial t = \partial \hat{X}/\partial t < 0, \partial \hat{P}/\partial t > 0 \), and \( \partial \hat{\rho}/\partial t > 0 \).

(ii) For a given entry cost \( K_H \), the numbers of firms \( \hat{m}, \hat{n} \) are decreasing in \( t \), while the relative market thickness \( \hat{z} \equiv \hat{n}/\hat{m} \) is increasing in \( t \); i.e., \( \partial \hat{m}/\partial t < 0, \partial \hat{n}/\partial t < 0 \) and \( \partial \hat{z}/\partial t > 0 \).

(iii) Let \( r^* \equiv \hat{\rho} - t \) denote the price received by a Foreign firm. Then, there exists \( \epsilon^* \in (0, 1) \) such that

\[
\frac{\partial r^*}{\partial t} \leq 0 \iff \frac{\partial \hat{\rho}}{\partial t} \leq 1 \iff \epsilon \geq \epsilon^*.
\]
Lemma 4.2(iii) says that an increase in tariffs improves the terms-of-trade, i.e., lowers $r^*$, if and only if the demand is concave. Recall from Lemma 3.1(iii) that when the market structure is exogenous, an increase in tariffs reduces $r^*$ for all logconcave demand functions ($\epsilon \geq -1$). When the market structure is endogenous, in contrast, an increase in tariffs reduces $r^*$ only for concave demand functions ($\epsilon \geq \epsilon^*$). Thus the terms-of-trade improvement is less likely in the endogenous market structure. The reason is explained as follows. Differentiating the implicit terms-of-trade $r^* = \hat{r} - t$ and using $\frac{\partial \hat{X}}{\partial t} = \hat{n} \frac{\partial \hat{x}}{\partial t} + \hat{x} \frac{\partial \hat{m}}{\partial t}$, we get

$$\frac{\partial r^*}{\partial t} = g_x(\hat{X}, \hat{m}) \hat{n} \frac{\partial \hat{x}}{\partial t} + g_x(\hat{X}, \hat{m}) \hat{x} \frac{\partial \hat{n}}{\partial t} + g_m(\hat{X}, \hat{m}) \frac{\partial \hat{m}}{\partial t} - 1,$$

where the second and third terms are absent in the exogenous market structure. This indicates that, when the market structure is endogenous, an increase in tariffs gives rise to additional adjustments through the exit of Home and Foreign firms. Further, substituting $\frac{\partial \hat{m}}{\partial t}$ and $\frac{\partial \hat{n}}{\partial t}$ in Lemma 4.2(ii) into the above expression, we get

$$\frac{\partial r^*}{\partial t} = g_x(\hat{X}, \hat{m}) \frac{\partial \hat{x}}{\partial t}.$$

Since $g_x(\hat{X}, \hat{m}) < 0$, the terms-of-trade improvement occurs ($\frac{\partial r^*}{\partial t} < 0$) if and only if tariffs raise average outputs of Foreign firms $\hat{x}$ ($\frac{\partial \hat{x}}{\partial t} > 0$). Although this is less likely to occur at first glance, we find that whether the average outputs $\hat{q}, \hat{x}$ decrease by tariff depends on the elasticity of slope of demand $\epsilon = \frac{Q'_P(Q)}{P'(Q)}$ in the endogenous market structure:

$$\frac{\partial \hat{q}}{\partial t} \gtrless 0 \iff \epsilon \gtrless 0, \quad \frac{\partial \hat{x}}{\partial t} \gtrless 0 \iff \epsilon \gtrless \epsilon^*,$$

which stands in sharp contrast to the exogenous market structure (see (3.7)). Intuitively, while an increase in tariffs decreases the aggregate outputs $\hat{Q}, \hat{X}$, it also decreases the number of firms $\hat{m}, \hat{n}$ in the respective market. Due to exit of rival firms, surviving firms might find it profitable to increase their outputs. More generally, our model suggests that the decrease in the aggregate outputs due to tariffs is largely accounted for by the decrease in the numbers of firms $\hat{m}, \hat{n}$, and net changes in the average outputs $\hat{q}, \hat{x}$ are ambiguous. This finding is in line with the recent literature of heterogeneous-firm trade models in which countries trade differentiated goods in monopolistic competition (see, e.g., Bernard et al. (2007)). We show that the similar result holds in the Cournot colocation model with free entry in which countries trade homogeneous goods in oligopolistic competition.

### 4.2 Tariffs

In the first stage, the Home government chooses a tariff rate $t$ to maximize Home welfare $W_H$, anticipating the output vector $(\hat{q}, \hat{Q}, \hat{x}, \hat{X})$, the price vector $(\hat{P}, \hat{r})$ and the number vector $(\hat{m}, \hat{n})$ in the Cournot competition. As profits are zero under free entry, Home welfare effectively consists
of consumer surplus and tariff revenue only. Thus, Home welfare is given by

\[ W_H = \sum_j \left[ \left( \int_0^Q_j P(y)dy - P(\hat{Q}_j)\hat{Q}_j \right) + t_j\hat{X}_j \right]. \]  \hspace{1cm} (4.8)

Differentiating \( W_H \) with respect to \( t_j \), we get

\[ \frac{dW_H}{dt_j} = \left( 1 - \frac{\partial P(\hat{Q}_j)}{\partial t_j} \right) \hat{Q}_j + t_j \frac{\partial \hat{X}_j}{\partial t_j}. \]  \hspace{1cm} (4.9)

Setting \( \frac{dW_H}{dt_j} = 0 \) and solving for \( t_j \) gives the expression for the optimal tariff which is presented later in Proposition 4.1. Noting \( \frac{\partial \hat{X}_j}{\partial t_j} < 0 \) in (4.9), the optimal tariff is strictly positive (negative) if and only if \( 1 - \frac{\partial P(\hat{Q}_j)}{\partial t_j} > (\leq) 0 \). In Section 3, we argued that a tariff induce the welfare loss due to the domestic distortion but the welfare gain from the terms-of-trade improvement. The above expression, however, is not directly related to how the terms-of-trade \( r_j^* \) improves by a tariff.

To better connect the optimal tariff in the short-run and long-run equilibria, note first from (4.2) that \( P(\hat{Q}_j)\hat{Q}_j = \hat{r}_j\hat{Q}_j + \hat{m}_jK_{H_j} \). Substituting this equality into (4.8) and using \( \hat{r}_j = g(\hat{X}_j, \hat{m}_j) \), Home welfare is written as

\[ W_H = \sum_j \left[ \int_0^{\hat{Q}_j} P(y)dy - g(\hat{X}_j, \hat{m}_j)\hat{Q}_j - \hat{m}_jK_{H_j} + t_j\hat{X}_j \right]. \]  \hspace{1cm} (4.10)

The expression in (4.10) implies that Home welfare is total surplus defined as the gross benefit to consumers less the sum of production costs and fixed entry costs (from Home’s perspectives). Using \( r_j^* \equiv \hat{r}_j - t_j \) and \( \hat{Q}_j = \hat{X}_j \), (4.10) is further rewritten as

\[ W_H = \sum_j \left[ \int_0^{\hat{Q}_j} P(y)dy - r_j^*\hat{X}_j - \hat{m}_jK_{H_j} \right]. \]

This expression is similar with that in the short run except for the extra term \( \hat{m}_jK_{H_j} \); in the long run, \( \hat{m}_j \) entering Home firms pay the fixed entry cost \( K_{H_j} \) and Home welfare takes into account the total entry cost \( \hat{m}_jK_{H_j} \). Differentiating this \( W_H \) with respect to \( t_j \), we get

\[ \frac{dW_H}{dt_j} = (P(\hat{Q}_j) - r_j^*) \frac{\partial \hat{Q}_j}{\partial t_j} - \frac{\partial r_j^*}{\partial t_j} \hat{X}_j - \frac{\partial \hat{m}_j}{\partial t_j} K_{H_j}. \]

As in the short run, the first and second terms respectively capture the welfare loss due to the domestic distortion and the welfare gain from the terms-of-trade improvement, but an increase in tariffs reduces \( r_j^* \) only for concave demand functions in the long run. The third term captures the welfare gain from the reduction in entry costs, since an increase in tariffs discourages entry of Home firms. Clearly, this welfare gain arises only in the long run.
Using the expression $\frac{\partial r^*_j}{\partial t_j} = \frac{\partial r_j}{\partial t_j} - 1$, we can express $\frac{dW_H}{dt_j}$ as follows:

$$
\frac{dW_H}{dt_j} = (P(Q_j) - \hat{r}_j) \frac{\partial \hat{Q}_j}{\partial t_j} + \left(1 - \frac{\partial \hat{r}_j}{\partial t_j}\right) \hat{X}_j - \frac{\partial \hat{m}_j}{\partial t_j} K_{H,j} + t_j \frac{\partial \hat{X}_j}{\partial t_j}.
$$

(4.11)

Since $\frac{\partial \hat{X}_j}{\partial t_j} < 0$ in (4.11), the optimal tariff is strictly positive (negative) if and only if

$$(P(Q_j) - \hat{r}_j) \frac{\partial \hat{Q}_j}{\partial t_j} + \left(1 - \frac{\partial \hat{r}_j}{\partial t_j}\right) \hat{X}_j - \frac{\partial \hat{m}_j}{\partial t_j} K_{H,j} > (\leq) 0.
$$

(4.12)

Moreover, since $\frac{\partial \hat{Q}_j}{\partial t_j} = \hat{m}_j \frac{\partial q_j}{\partial t_j} + \hat{q}_j \frac{\partial \hat{m}_j}{\partial t_j}$ and $(P(Q_j) - \hat{r}_j)\hat{q}_j = K_{H,j}$, (4.12) is rewritten as

$$(P(Q_j) - \hat{r}_j)\hat{m}_j \frac{\partial q_j}{\partial t_j} + \left(1 - \frac{\partial \hat{r}_j}{\partial t_j}\right) \hat{X}_j > (\leq) 0.
$$

(4.13)

The expression in (4.13) is very similar to that in (3.9), except that the number of Home firms is endogenous here. Thus, even in the long run where profits are driven to zero, the optimal tariff rate strikes a balance between the domestic distortion effect and the terms-of-trade effect. Note, however, that the impact of tariffs on average output $(\frac{\partial q_j}{\partial t_j})$ and the terms-of-trade $(1 - \frac{\partial \hat{r}_j}{\partial t_j} = -\frac{\partial r^*_j}{\partial t_j})$ depends crucially on the market structure.

In the exogenous market structure, an increase in tariffs reduces the average output $(\frac{\partial q_j}{\partial t_j} < 0)$ and improves the terms-of-trade $(\frac{\partial r^*_j}{\partial t_j} > 0)$ for all logconcave demand functions (Lemma 3.1). In the endogenous market structure, the signs of them depend on the elasticity of slope of demand (Lemma 4.2). As a result, the sign of the optimal tariff depends on the elasticity. To see this, consider first concave demand functions ($\epsilon_j \geq \epsilon_f^*$). Since $\frac{\partial q_j}{\partial t_j}$ and $-\frac{\partial r^*_j}{\partial t_j}$ are positive, (4.13) shows that the optimal tariff is positive. These terms are negative for convex demand functions ($\epsilon_j \leq 0$) and the optimal tariff is negative. For the special case of linear demand ($\epsilon_j = 0$), the optimal tariff is always negative. This suggests that there exists a cutoff $\epsilon_j^* \in (0, 1)(< \epsilon_f^*)$ at which the sign of the optimal tariff is determined: the optimal tariff is positive for $\epsilon_j > \epsilon_j^*$, whereas the optimal tariff is negative for $\epsilon_j < \epsilon_j^*$.

Having shown that the sign of the optimal tariff depends on $\epsilon_j$ in the endogenous market structure, we next turn to considering what characterizes the magnitude of the optimal tariff under $\epsilon_j > \epsilon_j^*$. As in Section 4.1, we focus on Home’s entry cost and suppose that $K_{H,j}$ varies across industries. From Lemma 4.1, an industry with larger entry costs accommodates a smaller number of Home firms $\hat{m}_j$ and Foreign firms $\hat{n}_j$. Furthermore, since such an industry entails the greater price-cost margin in the Home market $P(Q_j) - \hat{r}_j$ and the Foreign market $\hat{r}_j - c_j - t_j$, the Home government has more incentives to impose a higher tariff $\hat{t}_j$ (note that both terms are positive if $\epsilon_j > \epsilon_j^*$ in (4.13)). Thus, the Home optimal tariff is strictly increasing in $K_{H,j}$ in the endogenous market structure, because an increase in $K_{H,j}$ makes both the Home market and the Foreign market thinner.
Proposition 4.1 presents these findings and provides a sharper characterization.

**Proposition 4.1** Let $t_j^*$ denote the optimal tariff in industry $j$. At $t_j = t_j^*$, the following holds:

$$
\hat{t}_j = -\hat{Q}_j P'(\hat{Q}_j) \left( \frac{2(\hat{m}_j + \hat{n}_j)\epsilon_j + (\epsilon_j + 1)(\epsilon_j - 2)}{4\hat{m}_j\hat{n}_j} \right),
$$

where $\hat{Q}_j$ is the aggregate output evaluated at $t_j = \hat{t}_j$. Furthermore,

(i) There exists $\epsilon_j^{**} \in (0, 1)$ such that

$$
\hat{t}_j \geq 0 \iff \epsilon_j \geq \epsilon_j^{**}.
$$

(ii) $\hat{t}_j$ is monotonically increasing (decreasing) in $K_{Hj}$ and $K_{Fj}$ if $\epsilon_j > (<) \epsilon_j^{**}$.

As in the exogenous market structure, both the domestic distortion effect and the terms-of-trade effect characterize the Home optimal tariff in the endogenous market structure (see (4.13)). In the latter market structure, however, firm entry is constrained by entry costs and higher entry costs in Home discourage not only entry of Home firms, but also entry of Foreign firms due to the “firm-colocation” effect. As the Home market is thinner, the Foreign market is also thinner, and both the domestic distortion effect and the terms-of-trade effect are more significant because the two terms in (4.13) shift in same directions. This induces the Home government to set the higher tariff, the thinner the Home market and the Foreign market.

The above intuition immediately reveals the impact of the entry cost of Foreign firms $K_{Fj}$. As in $K_{Hj}$, an increase in $K_{Fj}$ reduces the number of Foreign firms $\hat{n}_j$ and Home firms $\hat{m}_j$, which in turn increases the Home optimal tariff $\hat{t}_j$ by making both the Home market and the Foreign market thinner. Thus, the Home optimal tariff $\hat{t}_j$ is strictly increasing in $K_{Fj}$, again due to the “firm-colocation” effect. This means that, if the numbers of firms $\hat{m}_j, \hat{n}_j$ and the quantities $\hat{q}_j, \hat{x}_j$ are variant to trade policy, the optimal tariff $\hat{t}_j$ is higher in industry $j$ where both the Home market and the Foreign market are thinner. From the fact that the optimal tariff is strictly increasing in both $K_{Hj}$ and $K_{Fj}$, we can alternatively characterize that the optimal tariff $\hat{t}_j$ in terms of the relative entry cost $k_j \equiv \frac{K_{Hj}}{K_{Fj}}$: the optimal tariff $\hat{t}_j$ is higher in industry $j$ where $k_j$ is relatively larger, or $k_j$ is relatively lower, giving rise to a U-shaped relationship between $k_j$ and $\hat{t}_j$ under $\epsilon_j > \epsilon_j^{**}$.

What should we make of the facts that (a) demand curvature matters for the sign of optimal tariffs and (b) the relationship between market thickness and optimal tariffs differs between the endogenous and exogenous market structures? Our reading of the literature suggests that, in terms of the dependence of optimal policy on demand curvature, our results have a similar flavor to some of the existing results in the trade literature. For example, the classic result that the sign of optimal tariffs in the presence of a Foreign monopoly depends on whether there is incomplete pass-through, which in turn depends on whether the demand curve is flatter than the marginal
As for the difference between the endogenous and exogenous market structures, our results are also in line with Horstmann and Markusen (1986) and Venables (1985), who have shown that in the single-stage oligopoly models, free entry can alter optimal trade policy due to the firm-delocation effect in horizontal specialization (though our result is due to the firm-colocation effect in vertical specialization). This point has recently been re-examined by Etro (2011) and Bagwell and Staiger (2012a, b) in the contexts of strategic trade policy and trade agreements respectively. We do not necessarily consider (b) as a shortcoming, because it is an empirical question to address whether the exogenous or endogenous market structure is more apt in reality. To answer this question, Section 5 will empirically explore the relationship between market thickness and optimal tariffs by exploiting the vertical linkage between Japan and China.

As an illustrative example, consider again the following class of inverse demand functions: $P(Q_j) = a_j - Q_j^b_j$ for which $\epsilon_j = b_j - 1$. Applying (4.14), the optimal tariff is given by

$$\tilde{t}_j = \left[ \frac{2(b_j - 1)(\hat{m}_j + \hat{n}_j) + b_j(b_j - 3)}{4\hat{m}_j\hat{n}_j + b_j(1 + b_j)[2(\hat{m}_j + \hat{n}_j) + b_j]} \right] (a_j - c_j)b_j.$$  

The optimal tariff $\tilde{t}_j$ is positive (negative) for any $\hat{m}_j, \hat{n}_j (> 1)$ if demand is concave (convex) with $b_j > 2$ ($b_j < 1$), implying that there exists $b_j^* \in (1, 2)$ at which the sign of $\tilde{t}_j$ is determined (note, if demand is linear ($b_j = 1$), the optimal tariff is negative). Further, in contrast to the exogenous market structure, this optimal tariff is characterized by $\hat{m}_j$ and $\hat{n}_j$ symmetrically. Since both $\hat{m}_j$ and $\hat{n}_j$ are strictly decreasing in $K_{H_j}$ and $K_{F_j}$, we have that $\tilde{t}_j$ is strictly increasing (decreasing) in $K_{H_j}$ and $K_{F_j}$ for $b_j > (\leq) b_j^*$. As a result, $\tilde{t}_j$ is higher, the greater or smaller is the relative entry cost $k_j$. Figure 4.3 illustrates the non-monotone relationship between $k_j$ and $\tilde{t}_j$. 

**Figure 4.3 – Non-monotonicity of $\tilde{t}$**
5 Evidence

This section assesses empirically the relevance of our theoretical predictions: (i) the impact of tariffs on the extensive and intensive margins; and (ii) the impact of market thickness on tariffs, both of which depend on market structures. We construct a unique industry-level trade dataset by exploiting the vertical linkage between Japan (Home) and China (Foreign). Section 5.1 reports the regression specifications, Section 5.2 discusses the data source and the key variables of the dataset, Section 5.3 presents the estimation results, and Section 5.4 describes robustness checks.

5.1 Specifications

Specifications for the extensive and intensive margins: The first hypothesis is concerned with the response of the extensive and intensive margins to exogenous tariff changes which is different between the exogenous and endogenous market structures. Our theory predicts that, although a decrease in Japan’s tariff rates increases China’s aggregate exports in both market structures, this is largely accounted for by the increase in the intensive (extensive) margin in the exogenous (endogenous) market structure. To test this hypothesis, let \( X_{jt} \), \( n_{jt} \), and \( x_{jt} \) denote China’s aggregate exports, the number of Chinese firms, and China’s average exports in industry \( j \) and year \( t \) respectively. Using the product-level trade data on China’s exports to Japan outlined later, we conduct the following regressions:

\[
\ln X_{jt} = \alpha_0 + \alpha_1 \ln(1 + \tau_{jt}) + \theta_j + \theta_t + \epsilon_{jt}, \quad (5.1)
\]

\[
\ln n_{jt} = \beta_0 + \beta_1 \ln(1 + \tau_{jt}) + \theta_j + \theta_t + \epsilon_{jt}, \quad (5.2)
\]

\[
\ln x_{jt} = \gamma_0 + \gamma_1 \ln(1 + \tau_{jt}) + \theta_j + \theta_t + \epsilon_{jt}, \quad (5.3)
\]

where \( \tau_{jt} \) is the simple average of Japan’s MFN ad valorem tariff rates, \( \theta_j \) is the product fixed effects, \( \theta_t \) is the year fixed effects, and \( \epsilon_{jt} \) is an error term. While we consider specific tariffs in Sections 3 and 4, the tariff dataset provides ad valorem tariffs only; however, as shown by Ara and Ghosh (2016), the similar results would hold between specific tariffs and ad valorem tariffs. We employ the fixed effects model for our estimations and include the two fixed effects to control for any product-specific and macroeconomic shocks. In light of Lemma 3.1, we hypothesize that \( \alpha_1 < 0, \beta_1 = 0, \gamma_1 < 0 \) in the exogenous market structure. By contrast, in light of Lemma 4.2, we hypothesize that \( \alpha_1 < 0, \beta_1 < 0, \gamma_1 \geq 0 \) and \( |\beta_1| > |\gamma_1| \) in the endogenous market structure.

We also test the hypothesis of the firm-colocation effect that occurs in the endogenous market structure. Our theory predicts that a decrease in Japan’s tariff rates encourages not only entry of Chinese exporters but also entry of Japanese importers into the vertically-related respective market, which in turn raises market thickness in China relative to Japan (see Lemma 4.2(ii)). To test this hypothesis, let \( m_{jt} \) and \( z_{jt} = n_{jt}/m_{jt} \) denote the number of Japanese importers and market thickness in China relative to Japan in industry \( j \) and year \( t \) respectively. Using the merged industry-level trade data between China and Japan outlined later, we conduct the...
following regression:

\[
\ln m_{jt} = \delta_0 + \delta_1 \ln(1 + \tau_{jt}) + \theta_t + \epsilon_{jt},
\]

\[
\ln z_{jt} = \eta_0 + \eta_1 \ln(1 + \tau_{jt}) + \theta_t + \epsilon_{jt}.
\]

While \(\delta_1 = 0\) and \(\eta_1 = 0\) in the exogenous market structure, we hypothesize that \(\delta_1 < 0\) and \(\eta_1 > 0\) in the endogenous market structure.

**Specifications for the impact of market thickness:** The second hypothesis is concerned with the impact of market thickness on Home optimal tariffs, which is also different between the exogenous and endogenous market structures. Our theory predicts that Home optimal tariffs are higher, the thicker (thinner) the Home market in the exogenous (endogenous) market structure. While export supply elasticities are usually employed to estimate optimal tariffs in the literature (e.g., Broda et al., 2008), market thickness plays a similar role with export supply elasticities in characterizing Home optimal tariffs, as shown in the theoretical sections. Thus, we make use of market thickness rather than export supply elasticities and employ the Herfindahl index (HHI) to measure market power in our regression strategy. Let \(HHI_{Hjt}\) and \(HHI_{Fjt}\) denote the HHI in industry \(j\) and year \(t\) in Japan and China respectively. Using the merged industry-level trade data between China and Japan, we conduct the following regression:

\[
\ln(1 + \tau_{jt}) = \zeta_0 + \zeta_1 HHJ_{Hjt} + \zeta_2 HHJ_{Fjt} + W'_{jt} \zeta_3 + \theta_t + \epsilon_{jt}.
\]

Suppose that a thicker market that accommodates a larger number of firms is more competitive. Then Proposition 3.1 suggests that Japan’s tariff rates are higher, the more competitive the Japanese market and the less competitive the Chinese market. Thus we hypothesize that \(\zeta_1 < 0\) and \(\zeta_2 > 0\) in the exogenous market structure. In contrast, Proposition 4.1 suggests that Japan’s tariff rates are higher, the less competitive both the Japanese market and the Chinese market. Thus we hypothesize that \(\zeta_1 > 0\) and \(\zeta_2 > 0\) in the endogenous market structure.

We include the number of employees and labor productivity in each industry of Japan and China to the vector \(W_{jt}\) in the regressions in order to control for the impact of market size and production efficiency on the characterization of Japan’s tariff rates.

**5.2 Data**

**Data on China’s exports to Japan:** Our dataset is the census of annual firm-level export and import transactions in China for the period from 2000 to 2009, collected by China Customs. The dataset contains trade value, quantity, and destination at 8-digit Harmonized System (HS) product classification. We use the publicly available concordance tables for 1997, 2002 and 2007 HS codes to make the product code consistent over time. The original dataset from firm-product-level is aggregated into the 6-digit HS product level to obtain the total number of exporters to
Japan (extensive margin) and their average export value (intensive margin) in thousand U.S. dollar. Our China Customs dataset covers a total of over 5,000 products at the 6-digit level from manufacturing industries.

To be consistent with theory, we need to focus only on homogeneous-input trade in (5.1)–(5.6). For this purpose, we divide our dataset into differentiated goods and homogeneous goods by the Rauch (1999) classification where Standard International Trade Classification (SITC) codes are converted into HS codes. (Commodities and reference-priced goods are treated as homogeneous goods.) We also divide our dataset into final goods and intermediate inputs by the classification of Broad Economic Categories (BEC). Thus, China’s exports to Japan are classified along with these two dimensions: whether China’s exports are differentiated goods or homogeneous goods and whether China’s exports are final goods or intermediate inputs. In 2005, for example, the numbers of homogeneous intermediate inputs and differentiated intermediate inputs are 1,073 and 1,331 respectively, while the numbers of homogeneous final goods and differentiated final goods are 183 and 1,310 respectively. These numbers show that the fraction of homogeneous goods is relatively large in China’s intermediate-input exports, compared to that of final-good exports. This would reflect the fact that China’s intermediate-input exports tend to be less skill and R&D intensive in vertical specialization (e.g., Dai et al., 2016).2

There is a well-known empirical regularity that intermediate-input trade is growing faster than final-good trade (Hummels et al. 2001; Hanson et al. 2005). To see this in our dataset, Table 5.1 presents China’s export growth rates in final-good exports and intermediate-input exports, decomposing the total exports into the extensive margin and the intensive margin. As shown in the first column, the export growth rate in intermediate-input trade is on average twice higher than that of final-good trade over years 2000–2008. The second and third columns further reveal

2For a cross-industry distribution among different types of trade, see Table C.1 in Appendix.

### Table 5.1 – China’s export growth rates from 2000 to 2009

<table>
<thead>
<tr>
<th>Year</th>
<th>Total</th>
<th>Extensive</th>
<th>Intensive</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>8%</td>
<td>5%</td>
<td>4%</td>
</tr>
<tr>
<td>2002</td>
<td>6%</td>
<td>12%</td>
<td>–7%</td>
</tr>
<tr>
<td>2003</td>
<td>18%</td>
<td>9%</td>
<td>8%</td>
</tr>
<tr>
<td>2004</td>
<td>18%</td>
<td>11%</td>
<td>8%</td>
</tr>
<tr>
<td>2005</td>
<td>12%</td>
<td>12%</td>
<td>0%</td>
</tr>
<tr>
<td>2006</td>
<td>4%</td>
<td>8%</td>
<td>–4%</td>
</tr>
<tr>
<td>2007</td>
<td>–6%</td>
<td>9%</td>
<td>–16%</td>
</tr>
<tr>
<td>2008</td>
<td>7%</td>
<td>1%</td>
<td>6%</td>
</tr>
<tr>
<td>2001–2008 avg</td>
<td>8%</td>
<td>8%</td>
<td>0%</td>
</tr>
<tr>
<td>2009</td>
<td>–4%</td>
<td>1%</td>
<td>–5%</td>
</tr>
</tbody>
</table>

(a) Final-good exports

<table>
<thead>
<tr>
<th>Year</th>
<th>Total</th>
<th>Extensive</th>
<th>Intensive</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>8%</td>
<td>10%</td>
<td>–2%</td>
</tr>
<tr>
<td>2002</td>
<td>11%</td>
<td>19%</td>
<td>–9%</td>
</tr>
<tr>
<td>2003</td>
<td>25%</td>
<td>18%</td>
<td>7%</td>
</tr>
<tr>
<td>2004</td>
<td>27%</td>
<td>18%</td>
<td>9%</td>
</tr>
<tr>
<td>2005</td>
<td>16%</td>
<td>13%</td>
<td>4%</td>
</tr>
<tr>
<td>2006</td>
<td>12%</td>
<td>10%</td>
<td>2%</td>
</tr>
<tr>
<td>2007</td>
<td>4%</td>
<td>14%</td>
<td>–10%</td>
</tr>
<tr>
<td>2008</td>
<td>17%</td>
<td>3%</td>
<td>14%</td>
</tr>
<tr>
<td>2001–2008 avg</td>
<td>15%</td>
<td>13%</td>
<td>2%</td>
</tr>
<tr>
<td>2009</td>
<td>–32%</td>
<td>2%</td>
<td>–35%</td>
</tr>
</tbody>
</table>

(b) Intermediate-input exports
Table 5.2 – Descriptive statistics on China’s export growth rates between 2000 and 2009

<table>
<thead>
<tr>
<th>Margin</th>
<th>No. of obs</th>
<th>Average</th>
<th>S.D.</th>
<th>Min</th>
<th>25th</th>
<th>Median</th>
<th>75th</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>29,755</td>
<td>15%</td>
<td>0.8</td>
<td>-334%</td>
<td>-18%</td>
<td>11%</td>
<td>43%</td>
<td>386%</td>
</tr>
<tr>
<td>Extensive</td>
<td>29,755</td>
<td>10%</td>
<td>0.39</td>
<td>-241%</td>
<td>-9%</td>
<td>8%</td>
<td>29%</td>
<td>349%</td>
</tr>
<tr>
<td>Intensive</td>
<td>29,755</td>
<td>5%</td>
<td>0.75</td>
<td>-429%</td>
<td>-27%</td>
<td>3%</td>
<td>35%</td>
<td>451%</td>
</tr>
</tbody>
</table>

(a) Total exports

<table>
<thead>
<tr>
<th>Margin</th>
<th>No. of obs</th>
<th>Average</th>
<th>S.D.</th>
<th>Min</th>
<th>25th</th>
<th>Median</th>
<th>75th</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>7,784</td>
<td>17%</td>
<td>0.92</td>
<td>-333%</td>
<td>-25%</td>
<td>13%</td>
<td>55%</td>
<td>386%</td>
</tr>
<tr>
<td>Extensive</td>
<td>7,784</td>
<td>10%</td>
<td>0.44</td>
<td>-204%</td>
<td>-14%</td>
<td>7%</td>
<td>34%</td>
<td>256%</td>
</tr>
<tr>
<td>Intensive</td>
<td>7,784</td>
<td>7%</td>
<td>0.86</td>
<td>-429%</td>
<td>-35%</td>
<td>6%</td>
<td>47%</td>
<td>427%</td>
</tr>
</tbody>
</table>

(b) Homogeneous intermediate-input exports

that the extensive margin has made a relatively greater contribution to this export growth than the intensive margin over our sample period, which is more predominant in intermediate-input trade than final-good trade.

Table 5.2 presents some descriptive statistics on the export growth rates of each margin. Here we report these statistics for the full sample (total exports) and the subsample (homogeneous intermediate-input exports). Note that the growth rates are higher for the subsample than the full sample; and the growth rates of the extensive margin is greater than the intensive margin for both the full sample and subsample. Comparing these with those in the existing literature, Feng et al. (2017) show that the contribution of the exporting firm number growth (i.e., extensive margin) is important in the growth rates of the total exports for China between 2000 and 2006, though they do not pay attention to the intensive margin and do not distinguish types of exports. In contrast, Buono and Lalanne (2012) show that the extensive (intensive) margin accounts for 40% (60%) of the growth rates of the total exports for France between 1994 and 2001.

Data on Japan’s imports from China: The dataset is from the Basic Survey of Business Structure and Activities (BSBSA), collected annually by Japan’s Ministry of Economy, Trade and Industry (METI) from 2000 to 2013. This national survey covers all firms with at least 50 employees or paid-up capital is over 30 million yen in manufacturing, mining and several service industries. We consider only manufacturing industries, which have about 10,000 firms each year. This dataset contains information on firm activities such as sales, purchases, exports, imports and 3-digit industry affiliation.

Before 2009, firms only report their exports and imports by major regional destination/origin, i.e., Asia, Middle East, Europe, Northern America, Africa and other regions. Fortunately, after 2009, firms are required to report whether they import from China. To implement our empirical analysis, we first sum the number of importers from Asia and the number of importers from
Table 5.3 – Descriptive statistics on Japan’s tariff rates on China’s exports in 2005

<table>
<thead>
<tr>
<th>Types of exports</th>
<th>No. of obs</th>
<th>Average</th>
<th>S.D.</th>
<th>Min</th>
<th>25th</th>
<th>Median</th>
<th>75th</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>All products</td>
<td>3,857</td>
<td>4.55</td>
<td>20.0</td>
<td>0</td>
<td>1.2</td>
<td>4.8</td>
<td>610.4</td>
<td></td>
</tr>
<tr>
<td>All intermediate inputs</td>
<td>2,364</td>
<td>3.51</td>
<td>18.0</td>
<td>0</td>
<td>1.5</td>
<td>3.9</td>
<td>610.4</td>
<td></td>
</tr>
<tr>
<td>Homogeneous intermediate inputs</td>
<td>1,055</td>
<td>4.99</td>
<td>26.0</td>
<td>0</td>
<td>3.1</td>
<td>4.4</td>
<td>610.4</td>
<td></td>
</tr>
</tbody>
</table>

Figure 5.1 – Tariff changes from 2000 to 2009

China at the 3-digit industry-level for the period of 2009 to 2013 respectively, and then calculate the five-year average share of importing from China to importing from Asia in each industry. From 2000 to 2008, we use this five-year average importer share to calculate the number of firms importing from China in each industry, since the dataset does not contain the number of importers from China during this period. We mainly conduct our empirical analysis at the 3-digit BSBSA industry-level.

Data on Japan’s tariff rates: The dataset of Japan’s tariff rates is obtained from the World Integrated Trade Solution (WITS) website. Specifically, we use the Trade Analysis Information System (TRAINS) dataset. For each product at the 6-digit HS level, the tariff dataset provides detailed information on tariff lines, average, minimum and maximum ad valorem tariff duties, and we use simple average MFN tariffs in our analysis.

Table 5.3 reports the descriptive statistics on Japan’s tariff rates on China’s exports in 2005 for the full sample (all products) and the subsamples (all intermediate inputs and homogeneous intermediate inputs). We find that, relative to other types of trade, homogeneous intermediate-input trade receives the higher tariff rates with the greater variations across industries. To show that this is not specific to the particular year, Figure 5.1 illustrates changes in Japan’s average tariff rate on China’s exports from 2000 to 2009. We find the similar trend (in terms of not only average but also standard deviation) in our sample period.
We recognize that the Japanese government does not optimally choose the tariff rates as our theory predicts. As argued by Broda et al. (2008), however, the insight that optimal tariffs are increasing in market power does not require governments to maximize welfare. It is also hard to believe that the Japanese government sets randomly the tariff rates without taking account of the trading environments to which Japanese firms belong. Given considerable tariff variations across industries and types of exports, the regression in (5.6) tries to capture these variations by market thickness that crucially affects market power.

**Merged industry-level China-Japan trade data:** Since the number of Japanese importers is calculated at the 3-digit BSBSA industry-level, we need to aggregate the dataset on the number of Chinese exporters, China’s average exports and Japan’s tariff rates at the HS-product level into those at the industry level. We first match the HS classification to International Standard Industrial Classification (ISIC) by using the concordance table from the World Integrated Trade Solution (WITS) website. We then match the ISIC classification to the Japan Standard Industrial Classification (JSIC) and BSBSA industry. For each industry and each year, we aggregate the number of Chinese exporters and calculate China’s average exports and Japan’s tariff rates. In the merged China-Japan trade data, every Chinese variable is converted into one million yen.

**Data on other independent variables:** Other independent variables – the HHI, the number of employees and labor productivity – come from three datasets.

The first dataset is the Annual Survey of Industrial Firms (ASIF), conducted by the National Bureau of Statistics of China for the period from 2000 to 2009. This is the most comprehensive and representative firm-level dataset in China, and surveyed firms contribute to the majority of China’s industrial output. These annual surveys cover all state-owned firms and other firms in industrial sectors with annual sales greater than 5 million yuan. The ASIF data provide detailed information, including 4-digit industry affiliation, sales, and the number of employees. We define labor productivity as the ratio of sales to the number of employees. The data on the number of employees and labor productivity at the firm level are aggregated into the industry level to use them as the control variables. As our analysis using the merged China-Japan trade data is at the industry level, we convert the Chinese industry codes (GB/T 4754-2002 classification) to the Japanese industry classifications and aggregate these control variables at the BSBSA industry level and convert these variables into one million yen.

The second one is the above China Customs data. We use these data to construct the industry-level HHI in China to measure the impact of market thickness. Specifically, we first calculate the HHI in China at the HS 6-digit product-year level and aggregate them to ISIC and then the BSBSA industry level.

The third one is the above BSBSA data from METI. These dataset are used to construct the industry-level HHI and the other controls (the number of employment and labor productivity) in Japan from the firm-level data. All these industry-level variables are merged at the BSBSA industry level to implement our analysis.
### Table 5.4 – Extensive and intensive margins in China data only

<table>
<thead>
<tr>
<th>Variable</th>
<th>Before WTO</th>
<th>After WTO</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln(1 + (\tau_{jt}))</td>
<td>(-0.238)</td>
<td>(-0.349)</td>
</tr>
<tr>
<td>Product fixed effects</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Year fixed effects</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>No. of observations</td>
<td>10,082</td>
<td>8,304</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.781</td>
<td>0.811</td>
</tr>
</tbody>
</table>

#### (a) Full sample

<table>
<thead>
<tr>
<th>Variable</th>
<th>Before WTO</th>
<th>After WTO</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln(1 + (\tau_{jt}))</td>
<td>(1.427)</td>
<td>(-0.491)</td>
</tr>
<tr>
<td>Product fixed effects</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Year fixed effects</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>No. of observations</td>
<td>1,778</td>
<td>8,304</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.939</td>
<td>0.811</td>
</tr>
</tbody>
</table>

#### (b) Before/after WTO

Note: Standard errors in brackets

\(*p < 0.10, **p < 0.05, ***p < 0.01\)

### 5.3 Estimation results

**Estimation results of the extensive and intensive margins:** Let us report the estimation results of (5.1)–(5.3) first, which we focus only on the Customs data on China’s exports to Japan. Panel (a) of Table 5.4 presents the impact of tariffs on the dependent variables in (5.1)–(5.3). The first three columns report the regressions of (5.1)–(5.3). The coefficients in the first and second columns are negative whereas the coefficient in the third column is positive, although only the coefficient in the second column is significant at the 1% level.\(^3\) This suggests that reductions in Japan’s tariff rates increase China’s aggregate exports, but this increase is largely accounted for by the increase in the number of Chinese exporters; in contrast, China’s average exports decrease by reductions in Japan’s tariff rates. While this is in line with the empirical literature (see Bernard et al. 2007), the difference is that the existing work mainly focuses on the impact of distances and we focus only on the impact of tariffs between two countries with fixed distances. In this sense, our framework is different from that employed in the so-called gravity equation, but the estimation results for the two margins are similar. These results support the prediction in the endogenous market structure.

---

\(^3\)Following Buono and Lalanne (2012), we do not consider lag in the regressions, but if we take one-period lag for Japan’s tariff rates, the coefficient in the first column is significant at the 10% level (see Table C.2 in Appendix).
Table 5.5 – Extensive and intensive margins in merged China-Japan data

<table>
<thead>
<tr>
<th></th>
<th>$\ln X_{jt}$</th>
<th>$\ln n_{jt}$</th>
<th>$\ln x_{jt}$</th>
<th>$\ln m_{jt}$</th>
<th>$\ln z_{jt}$</th>
<th>$\ln \bar{m}_{jt}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ln(1 + \tau_{jt})$</td>
<td>-0.371***</td>
<td>-0.515***</td>
<td>0.143</td>
<td>-0.320***</td>
<td>-0.195</td>
<td>-0.174***</td>
</tr>
<tr>
<td></td>
<td>(0.136)</td>
<td>(0.118)</td>
<td>(0.088)</td>
<td>(0.095)</td>
<td>(0.174)</td>
<td>(0.067)</td>
</tr>
<tr>
<td><strong>Year FE</strong></td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>No. of observations</strong></td>
<td>218</td>
<td>218</td>
<td>218</td>
<td>218</td>
<td>218</td>
<td>218</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.036</td>
<td>0.077</td>
<td>0.023</td>
<td>0.094</td>
<td>0.012</td>
<td>0.039</td>
</tr>
</tbody>
</table>

Note: Standard errors in brackets
*p < 0.10, **p < 0.05, ***p < 0.01

To examine whether China’s entry into the WTO caused a structural change, we divide the full sample into “before WTO” (2000–2001) and “after WTO” (2002–2009) and regress (5.1)–(5.3) within each subsample. Panel (b) of Table 5.4 presents these estimation results. Before entering the WTO, the coefficients in all columns are positive, though they are not statistically significant. After joining the WTO, however, the coefficient of the second column is statistically significant at the 1% level as in Panel (a). This difference implies that reductions in Japan’s tariff rates before joining the WTO have virtually no effect on China’s aggregate exports through both the margins, while these reductions after joining the WTO increase China’s aggregate exports particularly by inducing new Chinese firms to export into the Japanese markets. These results provide evidence that China’s entry to the WTO has changed the market structure in which the extensive margin plays a key role in China’s export growth.

Let us next report the estimation results of equations (5.4) and (5.5), which we use the merged industry-level trade data between China and Japan. Table 5.5 shows the impact of tariffs on the dependent variables in (5.1)–(5.5). Note that the coefficients of the first three columns are similar to those of Table 5.4(a), except that the coefficient of the first column is statistically significant at the 1% level. The coefficient in the fourth column is negative and statistically significant at the 1% level, which means that reductions in Japan’s tariff rates induce entry of Japanese firms. Noting that the coefficient of the second column is negative and statistically significant at the 1% level, this provides evidence of the firm-colocation effect in the data: reductions in Japan’s tariff rates increase not only the number of Chinese exporters but the number of Japanese importers in vertical specialization. Contrary to our theory, the coefficient of the fifth column is negative (although it is not statistically significant) and this would stem from our assumption of constant elasticity of slope. To examine whether tariff reductions give rise to the firm-colocation effect on the industry as a whole, we also regress (5.4) by replacing the total number of Japanese firms $\tilde{m}_{jt}$, which includes not only importing firms but also purely domestic non-importing firms as well. The coefficient of the sixth column is still negative and statistically significant at the 1% level, although it is smaller than that in the fourth column and the impact of tariffs on the extensive margin of non-importing firms is relatively weaker. Overall, we find that the estimation results are consistent with the prediction in the endogenous market structure.
Our finding that reductions in Japan’s tariff rates increase the extensive margin of China’s exports is similar to that in the previous study using the China Customs data (Feng et al., 2017). While the existing work focuses on the impact of China’s WTO accession on the extensive margin of China’s exports, we find that the response of the extensive margin to tariffs is significantly greater than the intensive margin after China’s WTO accession. More importantly, we provide evidence on the firm-colocation effect that China’s WTO accession encourages entry of Japanese firms as well as entry of Chinese firms. Thus, unilateral tariff reductions benefit both liberalizing and liberalized countries by driving the shift in the pattern of entry favoring these countries (i.e., the firm-colocation effect). This stands in contrast to horizontal specialization, where unilateral tariff reductions benefit a liberalized country but harm a liberalizing country by relocating firm entry from a liberalizing country to a liberalized country (i.e., the firm-delocation effect). Though we focus on homogeneous-good trade by identical firms, the firm-delocation effect also arises in differentiated-good trade by heterogeneous firms (see Melitz and Ottaviano, 2008) and we expect that the firm-colocation effect would arise in such a setting.

**Estimation results of the impact of market thickness:** Table 5.6 reports the estimates of equation (5.6). Panel (a) of Table 5.6 presents the impact of HHIs (a proxy of market thickness) on tariffs. Here China’s HHIs are calculated from the China Customs data that contain Chinese exporters only. The coefficients in the first column are positive, although only the coefficient on $HHI_{jt}$ is statistically significant at the 1% level. This suggests that Japan’s tariff rates are higher, the lower the degree of competition especially in Japan. The second column includes the number of employees in industries of each country to control for market size. The coefficient on $\ln Labor_{jt}$ is negative and significant at the 5% level; thus Japan’s tariff rates are higher, the smaller the industry size in Japan. This would probably reflect the aspect that industries with smaller scale easily form special interest groups to make pressures to the Japanese government for more protection from Chinese competition. In contrast, industry size of China has almost no impact on Japan’s tariff rates. The estimates of $\zeta_1$ and $\zeta_2$ remain positive and significant. In the third column, we also include labor productivity in industries of each country to control for production efficiency. The sign of the coefficients on $\ln LP_{jt}$ and $\ln LP_{F_{jt}}$ are opposite and both are statistically significant at the 1% level. Hence, Japan’s tariff rates are higher, the less (more) productive the importing (exporting) country, i.e., Japan (China). The estimates of $\zeta_1$ and $\zeta_2$ are still positive and the coefficient on $HHI_{jt}$ remain highly significant at the 5% level.

In Panel (b), China’s HHIs are calculated from the Annual Survey of Industrial Firms (ASIF), which contains not only exporting firms but also purely domestic non-exporting firms. The HHIs employed here do not overturn the estimates of $\zeta_1$ in each column, but make the estimates of $\zeta_2$ in the opposite signs which are statistically significant in the first and second columns. The differences in the estimates of $\zeta_2$ would reflect the aspect that the HHIs from the ASIF are less relevant to export supply elasticities and importer market power than the HHIs from the China Customs data, and they have a smaller impact on shaping Japan’s tariff rates. Nonetheless, the
### Table 5.6 – Tariffs and market thickness measured by Herfindahl indices

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln(1 + τ_{jt})</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$HHI_{Hjt}$</td>
<td>4.317***</td>
<td>2.619*</td>
<td>3.587**</td>
</tr>
<tr>
<td></td>
<td>(1.448)</td>
<td>(1.518)</td>
<td>(1.556)</td>
</tr>
<tr>
<td>$HHI_{Fjt}$</td>
<td>0.939</td>
<td>1.500**</td>
<td>0.536</td>
</tr>
<tr>
<td></td>
<td>(0.587)</td>
<td>(0.622)</td>
<td>(0.630)</td>
</tr>
<tr>
<td>ln Labor_{Hjt}</td>
<td>–0.169***</td>
<td>–0.180***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.082)</td>
<td>(0.072)</td>
<td></td>
</tr>
<tr>
<td>ln Labor_{Fjt}</td>
<td>0.034</td>
<td>0.041</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.053)</td>
<td>(0.048)</td>
<td></td>
</tr>
<tr>
<td>ln LP_{Hjt}</td>
<td>–0.520***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.137)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln LP_{Fjt}</td>
<td>0.703***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.138)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Year FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>No. of observations</td>
<td>218</td>
<td>218</td>
<td>218</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.073</td>
<td>0.094</td>
<td>0.209</td>
</tr>
</tbody>
</table>

(a) China Customs data

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln(1 + τ_{jt})</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$HHI_{Hjt}$</td>
<td>4.711***</td>
<td>3.936***</td>
<td>4.216***</td>
</tr>
<tr>
<td></td>
<td>(1.525)</td>
<td>(1.489)</td>
<td>(1.530)</td>
</tr>
<tr>
<td>$HHI_{Fjt}$</td>
<td>–0.985**</td>
<td>–1.144**</td>
<td>–0.757</td>
</tr>
<tr>
<td></td>
<td>(0.463)</td>
<td>(0.523)</td>
<td>(0.525)</td>
</tr>
<tr>
<td>ln Labor_{Hjt}</td>
<td>–0.062</td>
<td>–0.131*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.053)</td>
<td>(0.054)</td>
<td></td>
</tr>
<tr>
<td>ln Labor_{Fjt}</td>
<td>0.040</td>
<td>0.001</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.053)</td>
<td>(0.054)</td>
<td></td>
</tr>
<tr>
<td>ln LP_{Hjt}</td>
<td>–0.530***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.135)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln LP_{Fjt}</td>
<td>0.711***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.136)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Year FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>No. of observations</td>
<td>218</td>
<td>218</td>
<td>218</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.078</td>
<td>0.088</td>
<td>0.213</td>
</tr>
</tbody>
</table>

(b) Annual Survey of Industrial Firms

Note: Standard errors in brackets

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
result that $\zeta_1 > 0$ supports the prediction in the endogenous market structure and the result in Panel (a) provides indirect evidence on non-monotonicity of the optimal tariff.

The finding that Japan’s tariff rates are higher for industries where Japan’s market is thinner is similar to Broda et al. (2008) in that both analyze the role of market power in characterizing tariffs. In contrast to their study, we focus on market power arising from the domestic market as well as the export market in vertical specialization and employ market thickness rather than export supply elasticities. Although our theory identifies the similar role between them, we need to check whether our result also holds for these elasticities, which is left for future work.

5.4 Discussions

In this subsection, we describe several robustness checks of the main findings. First, we check whether it is crucial to distinguish processing trade from non-processing trade in our estimation. Second, we check whether our result holds not only for intermediate-input trade but also for total trade. To save space, we only report the estimation results of (5.1)–(5.3) with the full sample.

**Processing trade:** We first consider processing trade, which accounts for nearly half of China’s exports. As stressed by Dai et al. (2016), distinguishing between processing and non-processing trade is crucial for understanding China’s exports, since processing exporters are systematically different from non-processing exporters, i.e., processing exporters are less productive than non-processing exporters and non-exporters. To check whether the impact of tariff reductions on the extensive and intensive margins is different between these two types of exporters, we divide our dataset into processing and non-processing trade and regress (5.1)–(5.3) within each group.

Panel (a) of Table 5.7 presents the estimates of (5.1)–(5.3) where superscripts $N$ and $P$ are attached to the relevant variables of non-processing trade and processing trade respectively. For non-processing trade, the coefficients in each column are similar with those in Table 5.4(a), and thus tariff reductions induce new non-processing firms to export, but decrease the average non-processing exports. For processing trade, in contrast, the coefficients in each column are positive, although most of them are statistically insignificant. Therefore, tariff reductions have virtually no impact on each margin of these exports, or induce processing exporters to exit. The difference between processing and non-processing trade confirms the caveat raised by Dai et al. (2016): competition pressures due to tariff reductions outweigh cost reductions of reaching the export market for processing exporters who are less productive than other types of firms, which forces some of them to stop exporting. These results suggest our theoretical predictions mainly apply to non-processing exporters.

While we focus on processing trade, it is also well-known that state-owned-enterprises (SOEs) exhibit the similar property with processing exporters, in that SOEs are less productive than non-SOEs (e.g., Feng et al., 2017). We find that tariff reductions induce both new SOEs and new non-SOEs to export, but non-SOEs are more responsible to tariffs than SOEs (see Table C.3).
Table 5.7 – Robustness to processing trade and total trade

<table>
<thead>
<tr>
<th></th>
<th>$\ln X^N_{jt}$</th>
<th>$\ln n^N_{jt}$</th>
<th>$\ln X^P_{jt}$</th>
<th>$\ln n^P_{jt}$</th>
<th>$\ln x^N_{jt}$</th>
<th>$\ln x^P_{jt}$</th>
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</thead>
<tbody>
<tr>
<td>$\ln(1 + \tau_{jt})$</td>
<td>-0.263</td>
<td>-0.320***</td>
<td>0.057</td>
<td>0.400</td>
<td>0.159*</td>
<td>0.241</td>
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<td></td>
<td>(0.220)</td>
<td>(0.081)</td>
<td>(0.185)</td>
<td>(0.277)</td>
<td>(0.096)</td>
<td>(0.256)</td>
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<tr>
<td>Year FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
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<td>5,248</td>
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<tr>
<td>$R^2$</td>
<td>0.778</td>
<td>0.853</td>
<td>0.713</td>
<td>0.685</td>
<td>0.755</td>
<td>0.647</td>
</tr>
</tbody>
</table>

(a) Non-processing trade vs. processing trade

<table>
<thead>
<tr>
<th></th>
<th>$\ln X^I_{jt}$</th>
<th>$\ln n^I_{jt}$</th>
<th>$\ln x^I_{jt}$</th>
<th>$\ln X^T_{jt}$</th>
<th>$\ln n^T_{jt}$</th>
<th>$\ln x^T_{jt}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ln(1 + \tau_{jt})$</td>
<td>-0.094</td>
<td>-0.170***</td>
<td>0.076</td>
<td>-0.029</td>
<td>-0.094*</td>
<td>0.065</td>
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<tr>
<td></td>
<td>(0.167)</td>
<td>(0.064)</td>
<td>(0.135)</td>
<td>(0.120)</td>
<td>(0.048)</td>
<td>(0.099)</td>
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<tr>
<td>Product FE</td>
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<td>Yes</td>
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<td>Year FE</td>
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<td>No. of observations</td>
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<td>36,903</td>
<td>36,903</td>
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</tr>
<tr>
<td>$R^2$</td>
<td>0.810</td>
<td>0.902</td>
<td>0.705</td>
<td>0.836</td>
<td>0.925</td>
<td>0.723</td>
</tr>
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</table>

(b) Intermediate-input trade vs. total trade

Note: Standard errors in brackets

$p < 0.10, **p < 0.05, ***p < 0.01$

**Total trade:** We have so far restricted our attention to homogeneous intermediate-input trade. A natural question is whether the above finding also holds for different types of trade. To answer this question, we extend our analysis to other types of trade and compare the results.

Panel (b) of Table 5.7 presents the estimates of (5.1)–(5.3) where superscripts $I$ and $T$ are attached to the relevant variables of intermediate-input trade and total trade respectively. For intermediate-input trade, the coefficients in each column are similar with those in Table 5.4(a), though the estimated elasticities are smaller in the first and second columns. Thus, while tariff reductions give rise to the similar effect between differentiated and homogeneous intermediate-input trade, its impact is smaller for the differentiated inputs. For total trade, the similar pattern is observed, but the estimated elasticities in each column are smaller and the the significance level in the fifth column is lower. This implies that the impact of tariffs on the extensive margin is significantly weaker for final-good trade than intermediate-input trade. We are not aware of any studies that explore the different response to tariffs among the different types of trade, but this difference might be a key to understand why trade liberalization sometimes has few effects on the extensive margin. To assess how tariff reductions induce competition by encouraging entry of new exporters, the distinction is particularly important in the world of vertical specialization.

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4Broda et al. (2008) and Soderbery (2018) find that export supply elasticities for homogeneous goods are smaller than those for differentiated goods, but they do not consider differences between intermediate inputs and final goods.
6 Conclusion

With recent reductions in trade costs, firms from various countries are increasingly specializing in different but complementary stages of production sharing. In such environments of vertical specialization, under what conditions might a welfare maximizing government impose a tariff? We show that market thickness between vertically-related markets might be a crucial integrant for a domestic government to impose a tariff on foreign firms. In the exogenous market structure where the number of firms is fixed, we find that tariff reductions increases the intensive margin; and optimal tariffs are lower, the thinner the domestic market (relative to the foreign market). In the endogenous market structure where the number of firms is variant to trade policy, tariff reductions mainly increase the extensive margin; and optimal tariffs are lower, the thicker the domestic market. Constructing a unique industry-level trade dataset between Japan and China, we find evidence that supports the prediction in the endogenous market structure.

One of key policy implications is that unilateral tariff reductions benefit both liberalizing and liberalized countries in vertical specialization. Since final-good production relies on sequences of intermediate inputs that are fragmented across the globe, unilateral tariff reductions decrease the cost of production sharing and drive the shift in the pattern of entry favoring all countries. Given existing knowledge that unilateral tariff reductions benefit a liberalized country but harm a liberalizing country in horizontal specialization, our paper uncovers a new welfare gain from trade associated with unilateral tariff reductions in vertical specialization. Due to this channel, optimal tariffs are lower, the thicker the domestic market in the endogenous market structure where the extensive margin plays a key role. While we find evidence on these findings in vertical linkages between Japan and China, it should be noted that the same might not necessarily hold for different episodes of trade liberalization in other countries (e.g., Buono and Lalanne, 2012). If this is the case, tariffs have a few impacts on the extensive margin and the exogenous market structure would be more apt. Our model then indicates a different role of trade policy such that optimal tariffs are lower, the thinner the domestic market.

The structure of vertical oligopoly in our paper is admittedly simplistic. Each foreign firm has no choice but to sell intermediate inputs in spot markets, while each domestic firm has no choice but to purchase intermediate inputs in spot markets. In reality, however, domestic firms often also procure intermediate inputs through non-market mechanisms and negotiate with foreign firms over the terms of contracts in the global production chains. If we introduce the possibility of such contractual relationships in foreign outsourcing, market thickness should have an impact on optimal tariffs not only through the double-marginalization effect but also through matching and bargaining between domestic firms and foreign firms. While we have addressed the second option in a separate paper (Ara and Ghosh, 2016), a full-fledged analysis of trade policy that includes both contractual arrangements and spot markets is challenging. In the future work, we shall incorporate the interactions between these two options of foreign outsourcing to investigate its implication for trade policy.

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Appendix

A Proofs for Section 3

A.1 Equivalence between Assumptions 1 and 1’

The assumption \( Q(P) \) is logconcave implies

\[
\frac{d}{dP} \left[ \frac{d \ln Q(P)}{dP} \right] = \frac{d}{dP} \left[ \frac{Q'(P)}{Q(P)} \right] = \frac{Q(P) \cdot Q''(P) - [Q'(P)]^2}{[Q(P)]^2} \leq 0,
\]

which can be expressed as

\[
\frac{Q(P)Q''(P)}{[Q'(P)]^2} \leq 1. \tag{A.1}
\]

Differentiating \( P = P(Q(P)) \) with respect to \( P \), we get

\[ 1 = P'(Q(P))Q'(P). \]

Differentiating this once again with respect to \( P \) gives

\[ 0 = P''[Q'(P)]^2 + P'Q''(P). \]

Rewriting this equation, we get

\[
\frac{Q''(P)}{[Q'(P)]^2} = -\frac{P''}{P'}.
\]

Substituting this relationship into (A.1), we find that

\[-\frac{QP'''(Q)}{P''(Q)} \leq 1,
\]

which implies \( P'(Q) + QP''(Q) \leq 0. \)

\[ \square \]

A.2 Proof of Lemma 3.1

(i) Differentiating (3.6) with respect to \( n \), rearranging and using (3.6) subsequently, we get

\[
\frac{\partial \hat{X}}{\partial n} = -\frac{g(\hat{X}) - c - t}{(n + 1)g'(\hat{X}) + \hat{X}g''(\hat{X})} = \frac{\hat{x}}{n + 1 + \epsilon}.
\]

Note \( \hat{Q} = \hat{X} \) implies that \( \frac{\partial \hat{Q}}{\partial n} = \frac{\partial \hat{X}}{\partial n} \). Since \( \hat{Q} = m\hat{q} \) and \( \hat{X} = n\hat{x} \), we get

\[
\frac{\partial \hat{q}}{\partial n} = \frac{\hat{q}}{n(n + 1 + \epsilon)}, \quad \frac{\partial \hat{x}}{\partial n} = -\frac{(n + \epsilon)\hat{x}}{n(n + 1 + \epsilon)}.
\]
Using the expression for \( \frac{\partial \hat{X}}{\partial m} \) we get
\[
\frac{\partial \hat{r}}{\partial n} = \frac{\partial g(\hat{X})}{\partial n} = g'(\hat{X}) \frac{\partial \hat{X}}{\partial n} = \frac{\hat{x}g'(\hat{X})}{n + 1 + \epsilon} = \frac{\hat{x}P'(\hat{Q})(m + 1 + \epsilon)}{m(n + 1 + \epsilon)},
\]
\[
\frac{\partial \hat{P}}{\partial n} = \frac{\partial P(\hat{Q})}{\partial n} = P'(\hat{Q}) \frac{\partial \hat{Q}}{\partial n} = \frac{\hat{x}P'(\hat{Q})}{n + 1 + \epsilon}.
\]

The results follow from noticing that \( P'(\hat{Q}) < 0, m + 1 + \epsilon > 0 \) and \( n + 1 + \epsilon > 0 \).

(ii) Differentiating (3.6) with respect to \( t \), we get
\[
\frac{\partial \hat{X}}{\partial t} = \frac{n}{g'(\hat{X})(n + 1 + \epsilon)} = \frac{mn}{P'(\hat{Q})(n + 1 + \epsilon)(m + 1 + \epsilon)}.
\]

Note \( \hat{Q} = \hat{X} \) implies that \( \frac{\partial \hat{Q}}{\partial t} = \frac{\partial \hat{X}}{\partial t} \). Since \( \hat{Q} = m \hat{q} \) and \( \hat{X} = n \hat{x} \), we get
\[
\frac{\partial \hat{q}}{\partial t} = \frac{n}{P'(\hat{Q})(m + 1 + \epsilon)(n + 1 + \epsilon)}, \quad \frac{\partial \hat{x}}{\partial t} = \frac{m}{P'(\hat{Q})(m + 1 + \epsilon)(n + 1 + \epsilon)}.
\]

Using the expression for \( \frac{\partial \hat{X}}{\partial t} \) we get
\[
\frac{\partial \hat{r}}{\partial t} = g'(\hat{X}) \frac{\partial \hat{X}}{\partial t} = \frac{n}{n + 1 + \epsilon}, \quad \frac{\partial \hat{P}}{\partial t} = P'(\hat{Q}) \frac{\partial \hat{Q}}{\partial t} = \frac{mn}{(m + 1 + \epsilon)(n + 1 + \epsilon)}.
\]

The results follow from noticing that \( P'(\hat{X}) < 0, n + 1 + \epsilon > 0 \) and \( n + 1 + \epsilon > 0 \).

(iii) We have that
\[
\frac{\partial r^*}{\partial t} = \frac{\partial \hat{r}}{\partial t} - 1 = -\frac{1 + \epsilon}{n + 1 + \epsilon}.
\]

The claim follows from observing that \( n + 1 + \epsilon > 0 \).

Although we have focused on comparative statics with respect to \( n \), it is straightforward to examine comparative statics with respect to \( m \). From (3.1), we have that
\[
\frac{\partial \hat{Q}}{\partial m} = -\frac{P'(\hat{Q}) - r}{(m + 1)P'(\hat{Q}) + QP''(\hat{Q})} = \frac{\hat{q}}{m + 1 + \epsilon}.
\]

Since \( \hat{Q} = m \hat{q} \) and \( \hat{X} = n \hat{x} \), we get
\[
\frac{\partial \hat{q}}{\partial m} = -\frac{(m + \epsilon)\hat{q}}{m(m + 1 + \epsilon)}, \quad \frac{\partial \hat{x}}{\partial m} = \frac{\hat{x}}{m(m + 1 + \epsilon)}.
\]

Regarding the prices, note in particular that the input price \( r \) depends on \( m \) as well as \( X \) (see (3.2)). While we apply the short-hand definition \( r \equiv g(X) \) for the short-run analysis (since we mainly focus on comparative statics with respect to \( n \)), we need to explicitly define \( r \equiv g(X, m) \).
when we conduct comparative statics with respect to $m$. Thus

\[
\frac{\partial \hat{r}}{\partial m} = \frac{\partial g(\hat{X}, m)}{\partial m} = g_x(\hat{X}, m) \frac{\partial \hat{X}}{\partial m} + g_m(\hat{X}, m) = 0, \\
\frac{\partial \hat{P}}{\partial m} = \frac{\partial P(\hat{Q})}{\partial m} = P'(\hat{Q}) \frac{\partial \hat{Q}}{\partial m} = \frac{qP'(\hat{Q})}{m + 1 + \epsilon},
\]

where

\[
g_x(X, m) \equiv \frac{\partial g(X, m)}{\partial X}, \quad g_m(X, m) \equiv \frac{\partial g(X, m)}{\partial m}.\]

The results follow from noticing that $P'(\hat{Q}) < 0$ and $m + 1 + \epsilon > 0$. □

### A.2 Proof of Proposition 3.1

Setting $\frac{dW_H}{dt} = 0$ in (3.7) and rearranging, we immediately get (3.9). Concerning the properties of the optimal tariff $t = \hat{t}$, consider (i) first. It directly follows from (3.10) that

\[\hat{t} \geq 0 \iff n \leq \frac{n^*}{m + 1 + \epsilon} = (1 + \epsilon)(m + 1 + \epsilon).
\]

(ii) Differentiating $\frac{dW_H}{dt} \big|_{t=\hat{t}} = 0$ with respect to $n$ gives

\[
\frac{d\hat{t}}{dn} = -\frac{\frac{\partial^2 W_H}{\partial m \partial t}}{\frac{\partial^2 W_H}{\partial t^2}}.
\]

From the comparative statics results in Lemma 3.1, the second-order condition is satisfied, i.e.,

\[
\frac{\partial^2 W_H}{\partial t^2} = \left[ \frac{mn + (1 + \epsilon)(2 + \epsilon)(m + 1 + \epsilon)}{(m + 1 + \epsilon)(n + 1 + \epsilon)} \right] \frac{\partial \hat{Q}}{\partial t} < 0.
\]

Then it follows that

\[\text{sgn} \frac{d\hat{t}}{dn} = \text{sgn} \frac{\partial^2 W_H}{\partial n \partial t}. \quad \text{(A.2)}\]

Differentiating (3.7) with respect to $n$ gives

\[
\frac{\partial^2 W_H}{\partial n \partial t} = (1 + \epsilon) \left( -\frac{\hat{X}g'(\hat{X})}{n} \right) \left[ \frac{m + 2 + \epsilon}{(m + 1 + \epsilon)(n + 1 + \epsilon)} \right] \frac{\partial \hat{Q}}{\partial t} < 0.
\]

Since $\frac{\partial^2 W_H}{\partial m \partial t} < 0$, (A.2) implies that $\frac{d\hat{t}}{dn} < 0$.

We can also show that $t = \hat{t}$ is increasing in $m$. Differentiating (A.3) with respect to $m$ gives

\[
\frac{\partial^2 W_H}{\partial m \partial t} = \left( \frac{\hat{Q}P'(\hat{Q})}{m} \right) \left( \frac{1}{m + 1 + \epsilon} \right) \frac{\partial \hat{Q}}{\partial t} > 0.
\]

Since $\text{sgn} \frac{d\hat{t}}{dn} = \text{sgn} \frac{\partial^2 W_H}{\partial m \partial t}$, this implies that $\frac{d\hat{t}}{dm} > 0$. 41
While we focus on the sign of $\frac{\partial^2 W_{HT}}{\partial m \partial n}$ to show that $\frac{\partial i}{\partial n} < 0$, it is possible to show this by directly differentiating the optimal tariff in (3.10) with respect to $n$ and $m$. Using $P'(Q) = \frac{m}{m+1+\epsilon} g'(X)$ from (3.3), rewrite (3.11) as $\frac{\partial i}{\partial n} = -X g'(\hat{X}) \Phi$ where $\Phi \equiv \frac{(1+\epsilon)(m+1+\epsilon) - n}{(m+1+\epsilon)m}$. Differentiating this $\frac{\partial i}{\partial n}$ yields

$$\frac{\partial i}{\partial n} = \left[ g'(\hat{X}) + \hat{X} g''(\hat{X}) \right] \frac{\partial \hat{X}}{\partial n} \Phi - \hat{X} g'(\hat{X}) \frac{\partial \Phi}{\partial n}$$

Substituting $\frac{\partial \hat{X}}{\partial n}$ and $\frac{\partial \Phi}{\partial n}$ from Lemma 3.1(i)-(ii) and solving for $\frac{\partial i}{\partial n}$ yields

$$\frac{\partial i}{\partial n} = -(1+\epsilon) \left( -\frac{\hat{X} g'(\hat{X})}{n} \right) \left[ \frac{m + 2 + \epsilon}{mn + (1+\epsilon)(2+\epsilon)(m + 1 + \epsilon)} \right]. \tag{A.3}$$

The claim follows from noting that $g'(X) < 0$ and $1 + \epsilon > 0$. Following the similar steps, differentiating (3.10) with respect to $m$ yields

$$\frac{\partial i}{\partial m} = -\frac{\hat{Q} P'(\hat{Q})}{m} \left[ \frac{n + 1 + \epsilon}{mn + (1+\epsilon)(2+\epsilon)(m + 1 + \epsilon)} \right].$$

The claim follows from noting that $P'(Q) < 0$. \hfill \Box

### A.3 Proof of Proposition 3.2

We first show that the impact of $n$ on $\hat{\Pi}_F$ is decomposed into the competition effect and tariff-reduction effect. Differentiating $\hat{\Pi}_F = (\hat{r} - c - \hat{t}) \hat{X}$ with respect to $n$, we have that

$$\frac{\partial \hat{\Pi}_F}{\partial n} = (\hat{r} - c - \hat{t}) \frac{\partial \hat{X}}{\partial n} + \frac{\partial \hat{r}}{\partial n} \hat{X} - \frac{\partial \hat{t}}{\partial n} \hat{X}$$

$$= -\hat{x} g'(\hat{X}) \frac{\partial \hat{X}}{\partial n} + n \hat{x} g'(\hat{X}) \frac{\partial \hat{X}}{\partial n} - \frac{\partial \hat{t}}{\partial n} \hat{X}$$

$$= (n - 1) \hat{x} g'(\hat{X}) \frac{\partial \hat{X}}{\partial n} - \frac{\partial \hat{t}}{\partial n} \hat{X},$$

where the second equality comes from (3.6) and $\hat{X} = n \hat{x}$.

Next we show that the size effect can dominate the competition effect. Differentiating $\hat{\Pi}_F = (\hat{r} - c - \hat{t}) \hat{X} = -\frac{n^2 g'(X)}{n}$ with respect to $n$ gives

$$\frac{\partial \hat{\Pi}_F}{\partial n} = \frac{\hat{x}^2 g'(\hat{X})}{n^2} [1 - (2 + \epsilon) \phi], \tag{A.4}$$

where $\phi \equiv \frac{n}{\hat{X}} \frac{\partial \hat{X}}{\partial n} = \frac{n}{\hat{X}} \left( \frac{\partial \hat{X}}{\partial m} + \frac{\partial \hat{X}}{\partial t} \frac{\partial t}{\partial n} \right)$. Substituting $\frac{\partial \hat{X}}{\partial n}$ and $\frac{\partial \hat{X}}{\partial t}$ from Lemma 3.1(i)-(ii) and $\frac{\partial \hat{t}}{\partial n}$ from
(A.3) into \( \phi \), we have that
\[
\phi = \frac{1}{n + 1 + \epsilon} \left[ \frac{mn + (1 + \epsilon)(2 + \epsilon)(m + 1 + \epsilon) + n(1 + \epsilon)(m + 2 + \epsilon)}{mn + (1 + \epsilon)(2 + \epsilon)(m + 1 + \epsilon)} \right].
\]

Since \( \frac{\dot{X}^2 g'(\dot{X})}{n^2} < 0 \) in (A.4), \( \frac{\partial X}{\partial n} > 0 \) \( \iff \phi > \frac{1}{2 + \epsilon} \). Evaluating \( \lim_{m \to \infty} \phi \) and solving the last inequality for \( n \) establishes the result. Regarding per-firm Foreign profits \( \bar{\pi}_F \), we have that
\[
\frac{\partial \bar{\pi}_F}{\partial n} = \frac{\dot{X}^2 g'(\dot{X})}{n^3} \left[ 2 - (2 + \epsilon)\phi \right],
\]
which is always negative for \( \lim_{m \to \infty} \phi \), and the counter-intuitive outcome never occurs for \( \bar{\pi}_F \). \( \square \)

**B Proofs for Section 4**

**B.1 Proof of Lemma 4.1**

Let a dot represent proportional rates of change (e.g. \( \dot{Q} = \frac{Q'}{Q} \)) and totally differentiating (4.3), (4.2) and (4.4) respectively gives

\[
(n + 1 + \epsilon) \dot{X} = \left( \frac{n + 1 + \epsilon}{m + 1 + \epsilon} \right) \dot{m} + \dot{n}, \quad (B.1)
\]
\[
(2 + \epsilon) \dot{X} = 2\dot{m} + \dot{K}_H, \quad (B.2)
\]
\[
(2 + \epsilon) \dot{X} = \left( \frac{1 + \epsilon}{m + 1 + \epsilon} \right) \dot{m} + 2\dot{n}, \quad (B.3)
\]

where \( t \) and \( K_F \) hold constant. (B.1), (B.2) and (B.3) are the three equations that have the three unknowns \( \dot{X}(= \dot{Q}) \), \( \dot{m} \) and \( \dot{n} \), which can be solved explicitly as a function of \( \dot{K}_H \):

\[
\dot{X} = -\left( \frac{2n + 1 + \epsilon}{\Omega} \right) \dot{K}_H, \quad (B.4)
\]
\[
\dot{m} = -\left( \frac{(m + 1 + \epsilon)(2n + \epsilon)}{\Omega} \right) \dot{K}_H, \quad (B.5)
\]
\[
\dot{n} = -\left( \frac{n + 1 + \epsilon}{\Omega} \right) \dot{K}_H, \quad (B.6)
\]

where \( \Omega \equiv (2m + \epsilon)(2n + \epsilon) - (2 + \epsilon) > 0 \) for \( m > 1 \) and \( n > 1 \). Evaluated at \( X = \dot{X}, m = \dot{m}, n = \dot{n} \), we have that

\[
\frac{\partial \dot{X}}{\partial \dot{K}_H} = -\left( \frac{2n + 1 + \epsilon}{\Omega} \right) \frac{\dot{X} \dot{K}_H}{\dot{K}_H} < 0, \quad (B.4)
\]
\[
\frac{\partial \dot{m}}{\partial \dot{K}_H} = -\left( \frac{(m + 1 + \epsilon)(2n + \epsilon)}{\Omega} \right) \frac{\dot{m} \dot{K}_H}{\dot{K}_H} < 0, \quad (B.5)
\]
\[
\frac{\partial \dot{n}}{\partial \dot{K}_H} = -\left( \frac{n + 1 + \epsilon}{\Omega} \right) \frac{\dot{n} \dot{K}_H}{\dot{K}_H} < 0. \quad (B.6)
\]
Further, since $\dot{Q} = \dot{X}, \dot{Q} = \dot{m}q$ and $\dot{X} = \dot{n}x$, we have $\frac{\partial \dot{Q}}{\partial K_H} = \dot{m} \frac{\partial q}{\partial K_H} + \dot{q} \frac{\partial \dot{m}}{\partial K_H}$ and $\frac{\partial \dot{X}}{\partial K_H} = \dot{n} \frac{\partial x}{\partial K_H} + \dot{x} \frac{\partial \dot{n}}{\partial K_H}$, and using (B.4), (B.5) and (B.6) yields

$$\frac{\partial \dot{q}}{\partial K_H} = \left( \frac{\dot{m} + \epsilon (2\dot{n} + \epsilon) - 1}{\Omega} \right) \frac{\dot{q}}{K_H} > 0,$$

$$\frac{\partial \dot{x}}{\partial K_H} = - \left( \frac{\dot{n}}{\Omega} \right) \frac{\dot{x}}{K_H} < 0.$$

Using the expressions of $\frac{\partial \dot{X}}{\partial K_H}$ and $\frac{\partial \dot{m}}{\partial K_H}$ in (B.4) and (B.5), we also have that

$$\frac{\partial \dot{P}}{\partial K_H} = P'(\dot{Q}) \frac{\partial \dot{Q}}{\partial K_H} = - \left( \frac{2\dot{n} + 1 + \epsilon}{\Omega} \right) \frac{\dot{Q} P'(\dot{Q})}{K_H} > 0,$$

$$\frac{\partial \dot{r}}{\partial K_H} = g_x(\dot{X}, \dot{m}) \frac{\partial \dot{X}}{\partial K_H} + g_m(\dot{X}, \dot{m}) \frac{\partial \dot{m}}{\partial K_H} = - \left( \frac{1}{\Omega} \right) \frac{\dot{X} g_x(\dot{X}, \dot{m})}{K_H} > 0.$$

Next, differentiating $\dot{z}$ and $\frac{\partial \dot{P}}{\partial K_H}$ that are derived from (4.5) and (4.6) and using the expressions of $\frac{\partial \dot{X}}{\partial K_H}$ and $\frac{\partial \dot{m}}{\partial K_H}$ in (B.4) and (B.5), we have that

$$\frac{\partial \dot{z}}{\partial K_H} = \left( \frac{\dot{m} + \epsilon (2\dot{n} + \epsilon) + (\dot{n} - 1)}{\Omega} \right) \frac{\dot{z}}{K_H} > 0,$$

$$\frac{\partial \left( \frac{\partial \dot{P}}{\partial K_H} \right)}{\partial K_H} = \left( \frac{\dot{m} - 1 + \epsilon (\dot{n} - 1)}{\Omega} \right) \frac{k}{\dot{z} K_H} > 0.$$

These derivations establish the desired results.

It is straightforward to examine comparative statics with respect to $K_F$. Holding $t$ and $K_H$ constant, totally differentiating (4.3), (4.2) and (4.4) respectively gives

$$\frac{\partial \dot{X}}{\partial K_F} = - \left( \frac{2(\dot{m} + 1 + \epsilon)}{\Omega} \right) \frac{\dot{X}}{K_F} < 0,$$

$$\frac{\partial \dot{m}}{\partial K_F} = - \left( \frac{2 + \epsilon (\dot{m} + 1 + \epsilon)}{\Omega} \right) \frac{\dot{m}}{K_F} < 0,$$

$$\frac{\partial \dot{m}}{\partial K_F} = - \left( \frac{2\dot{m} + \epsilon (\dot{n} + 1 + \epsilon)}{\Omega} \right) \frac{\dot{m}}{K_F} < 0.$$

Note that the signs of $\frac{\partial \dot{X}}{\partial K_F}, \frac{\partial \dot{m}}{\partial K_F}$ and $\frac{\partial \dot{m}}{\partial K_F}$ are the same as those of $\frac{\partial \dot{X}}{\partial K_H}, \frac{\partial \dot{m}}{\partial K_H}$ and $\frac{\partial \dot{m}}{\partial K_H}$ respectively. Intuitively, this equivalence comes from the firm-colocation effect. The signs of $\frac{\partial \dot{P}}{\partial K_F}$ and $\frac{\partial \dot{r}}{\partial K_F}$ are also the same as those of $\frac{\partial \dot{P}}{\partial K_H}$ and $\frac{\partial \dot{r}}{\partial K_H}$:

$$\frac{\partial \dot{P}}{\partial K_F} = P'(\dot{Q}) \frac{\partial \dot{Q}}{\partial K_F} = - \left( \frac{2(\dot{m} + 1 + \epsilon)}{\Omega} \right) \frac{\dot{Q} P'(\dot{Q})}{K_F} > 0,$$

$$\frac{\partial \dot{r}}{\partial K_F} = g_x(\dot{X}, \dot{m}) \frac{\partial \dot{X}}{\partial K_F} + g_m(\dot{X}, \dot{m}) \frac{\partial \dot{m}}{\partial K_F} = - \left( \frac{2\dot{m} + \epsilon (\dot{n} + 1 + \epsilon)}{\Omega} \right) \frac{\dot{X} g_x(\dot{X}, \dot{m})}{K_F} > 0.$$
However, the signs of $\frac{\partial \hat{z}}{\partial K_F}$ and $\frac{\partial (\hat{P}_{c-t})}{\partial K_F}$ are opposite to those of $\frac{\partial \hat{z}}{\partial K_H}$ and $\frac{\partial (\hat{P}_{c-t})}{\partial K_H}$. Differentiating $\hat{z}$ and $\hat{P}_{c-t}$ that are derived from (4.5) and (4.6) and using the expressions of $\frac{\partial \hat{X}}{\partial K_F}$ and $\frac{\partial \hat{m}}{\partial K_F}$ above,

$$\frac{\partial \hat{z}}{\partial K_F} = -\left(\frac{\hat{m}(\hat{n} + \epsilon) + \hat{n}(\hat{m} + \epsilon) - 2(1 + \epsilon)}{\Omega}\right) \frac{\hat{z}}{K_F} < 0,$$

$$\frac{\partial (\hat{P}_{c-t})}{\partial K_F} = -\left(\frac{\hat{m}(\hat{n} + \epsilon) + \hat{n}(\hat{m} + \epsilon) + \epsilon(1 + \epsilon)}{\Omega}\right) \frac{k}{\hat{z}K_F} < 0.$$

The last comparative statics can be seen in Figure 4.5. As $K_F$ increases, only (4.6) shifts down while keeping (4.5) unchanged. As a result, both $\hat{z}$ and $\hat{P}_{c-t}$ decrease in the new equilibrium. In contrast to the case of $K_H$, the fact that $\hat{z}$ decreases with $K_F$ implies that, although both $\hat{m}$ and $\hat{n}$ are lowered by an increase in $K_F$, $\hat{n}$ declines relatively more than $\hat{m}$. □

**B.2 Proof of Lemma 4.2**

Using a dot representation once again (e.g. $\dot{Q} \equiv \frac{Q'}{Q}$) and totally differentiating (4.3), (4.2) and (4.4) respectively gives

$$(n + 1 + \epsilon)\dot{X} = \left(\frac{n + 1 + \epsilon}{m + 1 + \epsilon}\right) \dot{m} + \dot{n} + \frac{mnt}{(m + 1 + \epsilon)QP'(Q)} \dot{i},$$

$$\begin{align*}
(2 + \epsilon)\dot{Q} &= 2\dot{m}, \\
(2 + \epsilon)\dot{X} &= \left(\frac{1 + \epsilon}{m + 1 + \epsilon}\right) \dot{m} + 2\dot{n},
\end{align*}$$

where $K_H$ and $K_F$ hold constant. (B.8), (B.9) and (B.10) are three equations that have three unknowns $\dot{X}(= \dot{Q})$, $\dot{m}$ and $\dot{n}$, which can be solved explicitly as a function of $\dot{i}$:

$$\begin{align*}
\dot{X} &= \left(\frac{4}{\Omega}\right) \frac{mnt}{QP'(Q)} \dot{i}, \\
\dot{m} &= \left(\frac{2(2 + \epsilon)}{\Omega}\right) \frac{mnt}{QP'(Q)} \dot{i}, \\
\dot{n} &= \left(\frac{2 + \epsilon}{\Omega}\right) \left(\frac{2m + 1 + \epsilon}{m + 1 + \epsilon}\right) \frac{mnt}{QP'(Q)} \dot{i},
\end{align*}$$

where $\Omega$ is exactly the same as before. Evaluated at $X = \dot{X}, m = \dot{m}, n = \dot{n}$,

$$\begin{align*}
\frac{\partial \dot{X}}{\partial t} &= \left(\frac{4}{\Omega}\right) \frac{\dot{m}\dot{n}}{P'(Q)} < 0, \\
\frac{\partial \dot{m}}{\partial t} &= \left(\frac{2(2 + \epsilon)}{\Omega}\right) \frac{\dot{m}^2\dot{n}}{QP'(Q)} < 0, \\
\frac{\partial \dot{n}}{\partial t} &= \left(\frac{2 + \epsilon}{\Omega}\right) \frac{(2\dot{m} + 1 + \epsilon)\dot{n}^2}{X_g(X, \dot{m})} < 0.
\end{align*}$$
Further, since $\hat{Q} = \hat{X}$, $\hat{q} = \hat{m} \hat{q}$ and $\hat{X} = \hat{n} \hat{x}$, we have $\frac{\partial \hat{Q}}{\partial t} = \hat{n} \frac{\partial \hat{q}}{\partial t} + \hat{q} \frac{\partial \hat{n}}{\partial t}$ and $\frac{\partial \hat{X}}{\partial t} = \hat{n} \frac{\partial \hat{x}}{\partial t} + \hat{x} \frac{\partial \hat{n}}{\partial t}$, and using (B.11), (B.12) and (B.13) yields

\[
\frac{\partial \hat{q}}{\partial t} = -\frac{2\hat{n} \epsilon}{\Omega P'(Q)}, \tag{B.14}
\]

\[
\frac{\partial \hat{x}}{\partial t} = -\frac{2\hat{n} \epsilon + (\epsilon + 1)(\epsilon - 2)}{\Omega g_x(X, \hat{m})}, \tag{B.15}
\]

which suggests that

\[
\frac{\partial \hat{q}}{\partial t} \gtrless 0 \quad \iff \quad \epsilon \gtrless 0,
\]

\[
\frac{\partial \hat{x}}{\partial t} \gtrless 0 \quad \iff \quad \epsilon \gtrless \epsilon^*,
\]

where $\epsilon^* \in (0, 1)$ satisfies $2\hat{n} \epsilon^* + (\epsilon^* + 1)(\epsilon^* - 2) = 0$. Using the expressions of $\frac{\partial \hat{X}}{\partial t}$ and $\frac{\partial \hat{m}}{\partial t}$ in (B.11) and (B.12), we also have that

\[
\frac{\partial \hat{P}}{\partial t} = P'(\hat{Q}) \frac{\partial \hat{Q}}{\partial t} = \frac{4\hat{n} \hat{m}}{\hat{Q}} > 0,
\]

\[
\frac{\partial \hat{r}}{\partial t} = g_x(X, \hat{m}) \frac{\partial \hat{X}}{\partial t} + g_m(X, \hat{m}) \frac{\partial \hat{m}}{\partial t} = \frac{2\hat{n}(2\hat{m} + \epsilon)}{\hat{\Omega}} > 0.
\]

Next, differentiating $\hat{z}$ and $\frac{\hat{P} - \hat{r}}{\hat{r} - \epsilon - 1}$ that are derived from (4.5) and (4.6) and using the expressions of $\frac{\partial \hat{X}}{\partial t}$ and $\frac{\partial \hat{m}}{\partial t}$ in (B.11) and (B.12) and $k = \frac{n}{m + 1 + \epsilon}$ from (4.6), we have that

\[
\frac{\partial \hat{z}}{\partial t} = -\left(\frac{1 + \epsilon)(2 + \epsilon)}{\hat{\Omega}}\right) \frac{\hat{m} k}{Q P'(Q)} > 0, \tag{B.16}
\]

\[
\frac{\partial \left(\frac{\hat{P} - \hat{r}}{\hat{r} - \epsilon - 1}\right)}{\partial t} = \left(\frac{(1 + \epsilon)(2 + \epsilon)}{\hat{\Omega}}\right) \frac{\hat{m} k}{X g_x(X, \hat{m})} < 0.
\]

Finally, using the expression of $\frac{\partial \hat{r}}{\partial t}$, it directly follows that

\[
\frac{\partial \hat{r}^*}{\partial t} = \frac{\partial \hat{r}}{\partial t} - 1 = -\frac{2\hat{n} \epsilon + (\epsilon + 1)(\epsilon - 2)}{\hat{\Omega}}. \tag{B.17}
\]

Comparing (B.15) and (B.17) suggests that

\[
\frac{\partial \hat{r}^*}{\partial t} = g_x(X, \hat{m}) \frac{\partial \hat{x}}{\partial t}.
\]

The claim that $\frac{\partial \hat{r}^*}{\partial t} \leq 0 \iff \frac{\partial \hat{x}}{\partial t} \geq 0$ follows from noting that $g_x(X, m) < 0$. \qed
B.3 Proof of Proposition 4.1

Setting \( \frac{dW_t}{dt} = 0 \) in (4.10) and rearranging, we immediately get (4.14). Concerning the properties of the optimal tariff \( t = \hat{t} \), consider (i) first. It directly follows from (4.14) that

\[
\hat{t} \geq 0 \iff \epsilon \geq \epsilon^*, \tag{B.18}
\]

where \( \epsilon^* \in (0, 1) < \epsilon^* \) satisfies \( 2(\hat{m} + \hat{n})\epsilon^* + (\epsilon^* + 1)(\epsilon^* - 2) = 0 \).

To show (ii), using (4.2), let us rewrite (4.14) as

\[
\hat{t} = \left( \frac{K_H}{\Delta} \right) \Delta \text{ where } \Delta \equiv 2\hat{m}\epsilon(1 + \hat{z}) + (\epsilon + 1)(\epsilon - 2).
\]

Differentiating this \( \hat{t} \) with respect to \( K_H \) yields

\[
\frac{d\hat{t}}{dK_H} = \left( 1 - \frac{K_H}{\hat{z}} \frac{d\hat{z}}{dK_H} - \frac{K_H}{\hat{X}} \frac{d\hat{X}}{dK_H} + \frac{K_H}{\Delta} \frac{d\Delta}{dK_H} \right) \frac{\hat{t}}{K_H}. \tag{B.19}
\]

Noting that \( \frac{d\hat{z}}{dK_H} = \frac{\partial \hat{z}}{\partial K_H} + \frac{\partial \hat{t}}{\partial t} \frac{d\hat{z}}{dK_H} \) and others, rewrite the above expression as

\[
\left( 1 + \frac{\hat{t}}{\hat{z}} \frac{\partial \hat{z}}{\partial t} + \frac{\hat{t}}{\hat{X}} \frac{\partial \hat{X}}{\partial t} - \frac{\hat{t}}{\Delta} \frac{\partial \Delta}{\partial t} \right) \frac{d\hat{t}}{dK_H} = \left( 1 - \frac{K_H}{\hat{z}} \frac{\partial \hat{z}}{\partial K_H} - \frac{K_H}{\hat{X}} \frac{\partial \hat{X}}{\partial K_H} + \frac{K_H}{\Delta} \frac{\partial \Delta}{\partial K_H} \right) \frac{\hat{t}}{K_H}.
\]

Using (B.4), (B.5) and (B.7), we have that

\[
\frac{K_H}{\hat{X}} \frac{\partial \hat{X}}{\partial K_H} = -\left( \frac{2\hat{m} + 1 + \epsilon}{(\hat{m} + \epsilon)(2\hat{m} + \epsilon)} \right) \frac{K_H}{\hat{z}} \frac{\partial \hat{z}}{\partial K_H},
\]

\[
\frac{K_H}{\Delta} \frac{\partial \Delta}{\partial K_H} = -\frac{2\hat{m}\epsilon}{\Delta} \left( \frac{\hat{m} + 1 + \epsilon + \hat{z}}{\hat{m} + \epsilon} \right) \frac{K_H}{\hat{z}} \frac{\partial \hat{z}}{\partial K_H}.
\]

Similarly, using (B.11), (B.12) and (B.16), we have that

\[
\frac{\hat{t}}{\hat{X}} \frac{\partial \hat{X}}{\partial t} = -\left( \frac{\hat{m} + 1 + \epsilon}{(1 + \epsilon)(2 + \epsilon)} \right) \frac{\hat{t}}{\hat{z}} \frac{\partial \hat{z}}{\partial t},
\]

\[
\frac{\hat{t}}{\Delta} \frac{\partial \Delta}{\partial t} = -\frac{2\hat{m}\epsilon}{\Delta} \left( \frac{(2\hat{m} + 1 + \epsilon)(1 + \hat{z}) + 1 + \epsilon}{1 + \epsilon} \right) \frac{\hat{t}}{\hat{z}} \frac{\partial \hat{z}}{\partial t}.
\]

Substituting these into (B.19) and noting that \( \frac{K_H}{\hat{z}} \frac{\partial \hat{z}}{\partial K_H} < 1 \) (from (B.7)) and \( \frac{\hat{t}}{\hat{z}} \frac{\partial \hat{z}}{\partial t} < 1 \) (from (B.16)), we find that the value in the brackets of (B.19) is positive. Then, combining (B.18) and (B.19),

\[
\frac{d\hat{t}}{dK_H} \geq 0 \iff \epsilon \geq \epsilon^*.
\]

The similar proof applies to showing that \( t = \hat{t} \) is increasing in \( K_F \).
### Table C.1 – Industry share in China’s exports to Japan, 2005

<table>
<thead>
<tr>
<th>HS2 code</th>
<th>Industry</th>
<th>Homogeneous input</th>
<th>Differentiated input</th>
</tr>
</thead>
<tbody>
<tr>
<td>01–05</td>
<td>Animal products</td>
<td>0.4%</td>
<td>1.4%</td>
</tr>
<tr>
<td>06–15</td>
<td>Vegetable products</td>
<td>4.2%</td>
<td>3.0%</td>
</tr>
<tr>
<td>16–24</td>
<td>Food and beverage</td>
<td>2.2%</td>
<td>0.7%</td>
</tr>
<tr>
<td>25–27</td>
<td>Mineral</td>
<td>6.2%</td>
<td>2.0%</td>
</tr>
<tr>
<td>28–38</td>
<td>Chemical products</td>
<td>37.3%</td>
<td>8.3%</td>
</tr>
<tr>
<td>39–40</td>
<td>Plastic and rubber products</td>
<td>5.3%</td>
<td>7.6%</td>
</tr>
<tr>
<td>41–43</td>
<td>Leather products</td>
<td>0.2%</td>
<td>2.1%</td>
</tr>
<tr>
<td>44–49</td>
<td>Wood and pulp products</td>
<td>7.3%</td>
<td>4.5%</td>
</tr>
<tr>
<td>50–63</td>
<td>Textiles</td>
<td>18.9%</td>
<td>17.1%</td>
</tr>
<tr>
<td>64–67</td>
<td>Footwear and Headgear</td>
<td>0%</td>
<td>0.8%</td>
</tr>
<tr>
<td>68–71</td>
<td>Stone</td>
<td>0%</td>
<td>9.0%</td>
</tr>
<tr>
<td>72–83</td>
<td>Base metals</td>
<td>18.0%</td>
<td>14.7%</td>
</tr>
<tr>
<td>84–85</td>
<td>Machinery</td>
<td>0%</td>
<td>18.3%</td>
</tr>
<tr>
<td>86–89</td>
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</tr>
<tr>
<td>90–98</td>
<td>Parts and accessories</td>
<td>0%</td>
<td>7.7%</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>100%</td>
<td>100%</td>
</tr>
</tbody>
</table>

(a) Intermediate-input trade

<table>
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<th>Industry</th>
<th>Homogeneous final good</th>
<th>Differentiated final good</th>
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<tr>
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<td>1.5%</td>
</tr>
<tr>
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<td>Vegetable products</td>
<td>49.2%</td>
<td>0.4%</td>
</tr>
<tr>
<td>16–24</td>
<td>Food and beverage</td>
<td>21.9%</td>
<td>4.0%</td>
</tr>
<tr>
<td>25–27</td>
<td>Mineral</td>
<td>1.0%</td>
<td>0%</td>
</tr>
<tr>
<td>28–38</td>
<td>Chemical products</td>
<td>1.0%</td>
<td>3.0%</td>
</tr>
<tr>
<td>39–40</td>
<td>Plastic and rubber products</td>
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<td>1.0%</td>
</tr>
<tr>
<td>41–43</td>
<td>Leather products</td>
<td>0%</td>
<td>1.7%</td>
</tr>
<tr>
<td>44–49</td>
<td>Wood and pulp products</td>
<td>0%</td>
<td>2.7%</td>
</tr>
<tr>
<td>50–63</td>
<td>Textiles</td>
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<td>23.2%</td>
</tr>
<tr>
<td>64–67</td>
<td>Footwear and Headgear</td>
<td>0%</td>
<td>3.4%</td>
</tr>
<tr>
<td>68–71</td>
<td>Stone</td>
<td>0.5%</td>
<td>1.8%</td>
</tr>
<tr>
<td>72–83</td>
<td>Base metals</td>
<td>0%</td>
<td>6.1%</td>
</tr>
<tr>
<td>84–85</td>
<td>Machinery</td>
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<td>31.7%</td>
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<td>2.9%</td>
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<tr>
<td>90–98</td>
<td>Parts and accessories</td>
<td>0%</td>
<td>16.6%</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>100%</td>
<td>100%</td>
</tr>
</tbody>
</table>

(b) Final-good trade
### Table C.2 – One-period lag

<table>
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<th>(\ln n_{jt})</th>
<th>(\ln x_{jt})</th>
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<tbody>
<tr>
<td>(\ln(1 + \tau_{jt-1}))</td>
<td>-0.419* (0.224)</td>
<td>-0.378*** (0.089)</td>
<td>-0.041 (0.176)</td>
</tr>
<tr>
<td>Product FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Year FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>No. of observations</td>
<td>8,334</td>
<td>8,334</td>
<td>8,334</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.798</td>
<td>0.871</td>
<td>0.735</td>
</tr>
</tbody>
</table>

### Table C.3 – Non-SOEs vs. SOEs

<table>
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<tr>
<th></th>
<th>(\ln X_{jt}^N)</th>
<th>(\ln n_{jt}^N)</th>
<th>(\ln x_{jt}^N)</th>
<th>(\ln X_{jt}^S)</th>
<th>(\ln n_{jt}^S)</th>
<th>(\ln x_{jt}^S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\ln(1 + \tau_{jt}))</td>
<td>-0.238 (0.211)</td>
<td>-0.292*** (0.081)</td>
<td>0.054 (0.174)</td>
<td>-0.062 (0.223)</td>
<td>-0.263*** (0.082)</td>
<td>0.202 (0.184)</td>
</tr>
<tr>
<td>Product FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
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</tr>
<tr>
<td>Year FE</td>
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<td>Yes</td>
<td>Yes</td>
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<td>No. of observations</td>
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<td>(R^2)</td>
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<td>0.705</td>
<td>0.745</td>
<td>0.832</td>
<td>0.677</td>
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Note: Superscripts \(N\) and \(S\) to non-SOE trade and SOE trade
**References**


