Competition, Productivity and Trade, Reconsidered*

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Abstract

We develop a C.E.S. model of monopolistic competition with heterogeneous firms in which toughness of competition is endogenously adjusted by wages. We show that endogenous wages give rise to the pricing and price-cost margins that vary across markets and countries. As a result, not only does trade liberalization but country size also has an impact on firm selection. Operating through this channel, our model yields a set of predictions that are consistent with recent non-C.E.S. models but cannot be obtained in standard C.E.S. models.

Keywords: Trade liberalization, country size, productivity, home market effect, wages
JEL Classification Numbers: F12, F14

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1 Introduction

C.E.S. preferences and monopolistic competition are a workforce model in the new trade theory literature. One of the crucial drawbacks in this framework is that firms’ markups are constant and exogenously fixed by the symmetric elasticity of substitution among varieties, which in turn implies under firm heterogeneity that country size has no selection effects. To circumvent these unrealistic results in a C.E.S. model, the recent trade literature drops this preference assumption in order to allow for the variability of markups in response to toughness of competition across markets and countries. This paper takes another approach, endogenous wages, to fix the problem under the assumption of C.E.S. preferences. We show that our C.E.S. model can account for many of empirical facts that are demonstrated by a non-C.E.S. model, while preserving the usefulness of the workforce model in the new trade theory literature.

Our starting point is to notice that, even under C.E.S. preferences with the constant markups (defined as the price over the cost), the price-cost margins (defined as the price minus the cost) are not constant if wages are endogenously determined. Formally, letting \( p(\varphi) \) and \( c(\varphi) \) denote the (fob) price and cost of a firm with productivity \( \varphi \), it follows immediately from the pricing rule that the price-cost margins are given by

\[
p(\varphi) - c(\varphi) = \frac{1}{\sigma - 1} \frac{w}{\varphi};
\]

where \( \sigma \) is the elasticity of substitution and \( w \) is the wage rate. The price-cost margins vary with the degree of competitiveness measure (such as country size and trade liberalization) so long as wages are endogenously affected by these factors, even though the markups are constant due to C.E.S. preferences. Observe that the channel is not operative if wages are exogenously fixed by a freely tradable “outside” good, in which case not only are the markups but the price-cost margins are also constant under C.E.S. preferences. The purpose of this paper is to reconsider the role of country size in pinning down wages and trade patterns in increasing returns and monopolistic competition (known as the home market effect) a la Krugman (1980) and its impact on aggregate productivity a la Melitz (2003).

It is well-known that a country with larger size has higher wages in the presence of increasing returns to scale and variable trade costs, which gives rise to trade patterns such that the number of exporting firms increases more than proportionately to country size in the homogeneous-firm model (Krugman, 1980). This paper finds, in the heterogeneous-firm model, that the impact of country size on wages holds, whereas the impact on trade patterns does not hold. The impact of country size on wages helps explain empirical evidence documented in previous work even with C.E.S. preferences. For example, from (1) and the home market effect on wages, we can obtain the variable price-cost margins that are higher in a richer country of higher per capita income (Alexandria and Kaboski, 2011; Fieler, 2011). Moreover, since the result implies that toughness of competition varies with country size through wages, we have that some (less productive) firms
enter in economic booms when wages increase but they exit in recessions when wages decrease (Nerarda and Ramey, 2013). These findings are in line with those obtained in non-C.E.S. models, which cannot come without endogenous wages in standard C.E.S. models.

To understand the difference in trade patterns between the homogeneous and heterogeneous-firm models, it is useful to consider the role of an outside good. With an outside good that makes wages exogenously fixed and independent of country size, the difference in country size induces a larger (smaller) country to specialize in a differentiated (outside) good and to be a net exporter of the respective good. Since both markups and price-cost margins are exogenously fixed, however, country size has no impact on toughness of competition under C.E.S. preferences. As a result, changes in welfare operate through changes in trade patterns in the presence of an outside good in such a way that a country with larger (smaller) size gains (losses) from trade liberalization in the homogeneous-firm model (Krugman, 1980; Venables, 1987). Furthermore, this influence of country size on trade patterns and welfare also goes through in the heterogeneous-firm model (Melitz and Redding, 2014).

Without an outside good that makes wages endogenously determined and positively affected by country size, in contrast, the difference in country size induces a larger (smaller) country to exhibit higher (lower) price-cost margins (see (1)). Since firms face less competitive pressures in a larger country, less productive firms find it possible to survive (i.e., a “domestic productivity cutoff” falls), while simultaneously only more productive firms find it possible to export (i.e., an “export productivity cutoff” rises) there. Thus the number of exporting firms increases less than proportionately to that of domestic firms, violating the home market effect on trade patterns in the absence of an outside good in the heterogeneous-firm model. While a larger country exhibits lower productivity (associated with higher price-cost margins), this country nevertheless enjoys higher welfare gains because a negative impact on declined productivity is always dominated by a positive impact on increased product variety. Therefore our model features the welfare gains from trade highlighted by Krugman (1980).

Our model can offer the rationale why firms adopt different pricing and price-cost margins, and earn different revenues in different export markets as observed in various empirical data. In particular, there are lots of work to suggest that, even in the same variety, pricing and markups are positively correlated with wages in destination countries (Alexandria and Kaboski, 2011). To account for this stylized fact, the existing literature develops trade models based on monopolistic competition without C.E.S. preferences (e.g., Bertoletti and Etro, 2017; Di Comité et al., 2017). We show, however, that endogenous wages lead to a similar outcome even with a C.E.S. model. Our model predicts that firms export their goods at higher (lower) pricing and price-cost margins in a richer (poorer) country, which results in higher (lower) revenues obtained there. Moreover, the difference in revenues induces entry of firms that is no longer proportional to country size, which is characterized by concentration (expansion) of exporting firms in a rich (poor) country.

\(^1\)In the homogeneous-firm model where the number of exporting firms is the same as that of domestic firms, the home market effect on trade patterns is operative even in the absence of an outside good (Krugman, 1980).
Our finding for the impact of competition on productivity and welfare under the assumptions of C.E.S. preferences and monopolistic competition is similar to that in Demidova and Rodríguez-Clare (2013). They find that endogenous wages in the heterogeneous-firm model can reverse the impact of trade liberalization on welfare in a unilaterally liberalizing country because the home market effect is not operative on trade patterns without an outside good. While they show this with a simple figure that summarizes equilibrium relationships, we analytically show a similar result adopting the technique known as the exact hat algebra.\(^2\) A more important difference is however that in contrast to their work in which country size is used to obtain the equilibrium outcome for a small economy, we examine the impact of country size on wages and trade patterns as in Krugman (1980). This channel can account for the pricing, price-cost margins and revenues that endogenously vary across markets and countries, as those in non-C.E.S. models, which fits well with empirical evidence. As observed by Krugman (2008), wages are one of the most striking aspects of recent trade flows and we believe that our model can provide a different lens through which to understand the real world especially when toughness of competition is endogenously adjusted by wages.

The impact of competition on productivity and welfare has been extensively examined in the literature of non-C.E.S. preferences. Melitz and Ottaviano (2008) are the first to investigate pro-competitive effects of market size and trade in the heterogeneous-firm model under quasi-linear-quadratic preferences. They show that a country with larger size entails higher productivity and welfare through tougher competition in the domestic market, reducing firms’ average markups. Due to an outside good that gives rise to the home market effect on trade patterns, competition pressures from trade liberalization can result in a welfare loss from production relocation toward a higher trade cost country. We show, without an outside good, that the negative welfare effect is driven by changes in toughness of competition, but competition pressures from trade as well as market size are always welfare enhancing because the home market effect is not operative on trade patterns, just as in Demidova and Rodríguez-Clare (2013).

Finally, this paper is related to recent work that tries to explain different pricing, markups, and revenues of the same variety in different destination countries in monopolistic competition. For example, using an additively separable indirect utility, Bertoletti and Etro (2017) show that firms adopt higher pricing and markups in a richer country of higher per capita income. Using a quadratic utility, Di Comité et al. (2017) also show that an export price is lower when goods are shipped to a larger country with more intense competition which lowers residual demands and puts downward pressure on that price. While they drop C.E.S. preferences to derive their model outcome, we retain this preference assumption but allow toughness of competition to be adjusted wages endogenously. In so doing, we propose an alternative view of trade in which a difference in pricing, markups, and revenues of the same variety comes from the supply side (i.e., wages), rather than the demand side (i.e., preferences).

\(^2\)See Ossa (2016) for a recent survey using this technique that applies for the equilibrium analysis with endogenous wages.
2 Model

2.1 General Setup

Consider the Melitz (2003) model with \( N \) countries and \( S + 1 \) sectors. Country \( i \) is populated by a mass \( L_i \) of identical consumers whose preferences are

\[
U_i = \sum_{s=0}^{S} \mu_s \ln Q_{is}, \quad 0 < \mu_s < 1.
\]

Sector \( s = 0 \) is a homogeneous good, which is produced one-to-one with labor and freely tradable. The remaining \( s \geq 1 \) sectors are differentiated goods, and preferences are represented by a C.E.S. Dixit-Stiglitz form:

\[
Q_{is} = \left( \sum_{n=1}^{N} \int_{\omega_{ns} \in \Omega_{ns}} q_{ins}(\omega_{ns})^{\frac{\sigma_s-1}{\sigma_s}} d\omega_{ns} \right)^{\frac{\sigma_s}{\sigma_s-1}}, \quad \sigma_s > 1.
\]

As is well-known, the preferences yield the demand for a firm with variety \( \omega \) and a charging price \( p_{ijs}(\omega) \) from country \( i \) to country \( j \) in sector \( s(\geq 1) \):

\[
q_{ijs}(\omega) = R_{js} P_{js}^{\sigma_s-1} p_{ijs}(\omega)^{-\sigma_s},
\]

where \( R_{js} \) is aggregate expenditure and \( P_{js} \) is the price index of country \( j \) in sector \( s \). The upper tier Cobb-Douglas preferences imply that consumers spend \( R_{js} = \mu_s w_j \tilde{L}_j \) on goods produced by sector \( s \) (as aggregate income of country \( j \) is \( w_j \tilde{L}_j \) due to free entry).

In each of the differentiated-good sectors, a mass \( M_{is}^e \) of monopolistically competitive firms randomly draw productivity \( \varphi \) from a fixed distribution \( G_{is}(\varphi) \) upon paying a fixed entry cost \( f_{is}^e \) (measured in country \( i \)'s labor units). When a firm from country \( i \) chooses to serve for country \( j \), it pays a variable trade cost \( \tau_{ijs} > 1 \) (with \( \tau_{iis} = 1 \)) and a fixed trade cost \( f_{ijs} \) (both measured in country \( i \)'s labor units) where these costs satisfy \( \tau_{ijs}^{\sigma_s-1} f_{ijs} > f_{iis} \). It is useful to define

\[
J_{is}(\varphi^*) = \int_{\varphi^*}^{\infty} \left( \left( \frac{\varphi}{\varphi^*} \right)^{\sigma_s-1} - 1 \right) dG_{is}(\varphi),
\]

\[
V_{is}(\varphi^*) = \int_{\varphi^*}^{\infty} \varphi^{\sigma_s-1} dG_{is}(\varphi),
\]

where \( J_{is}(\varphi^*) \) and \( V_{is}(\varphi^*) \) are strictly decreasing in \( \varphi^* \).

In what follows, we explore the equilibrium characteristics in the following two cases. First, \( \mu_0 \) is large enough so that all countries produce an outside good, which makes wages exogenously fixed. Second, \( \mu_0 = 0 \) so that an outside good is absent, in which case wages are endogenously determined by the trade balance condition. As stressed in the Introduction, this comparison is made to examine the role of endogenous factoral terms of trade in monopolistic competition.
2.2 Equilibrium Conditions

From the pricing rule used in (1), firm revenue with productivity $\varphi$ from country $i$ to country $j$ is

$$r_{ijs}(\varphi) = \sigma_s B_{js}(\tau_{ijs}w_i)^{1-\sigma_s} \varphi^{\sigma_s-1},$$

where

$$B_{js} = \frac{(\sigma_s - 1)^{\sigma_s-1}}{\sigma_s} R_{js} P_{js}^{\sigma_s-1}$$

is the index of market demand. Noting that variable profit is $r_{ijs}(\varphi)/\sigma_s$, the productivity cutoff that satisfies zero profit ($r_{ijs}(\varphi_{ijs}^*) = w_i f_{ijs}$) is implicitly defined as

$$B_{js}(\tau_{ijs}w_i)^{1-\sigma_s}(\varphi_{ijs}^*)^{\sigma_s-1} = w_i f_{ijs}.$$ (2)

Free entry requires that the expected profits of entering the market in all operating countries equal the fixed entry costs ($\sum_n \int_{\varphi_{in}^c}^{\varphi_{ins}} \frac{r_{ins}(\varphi)}{\sigma_s} - w_i f_{ins}) dG_{i\varphi} = w_i f_{ins}^e$). Using the definition of $J_{is}(\cdot)$ in section 2.1, the free entry condition in country $i$ is

$$\sum_{n=1}^N f_{ins} J_{is}(\varphi_{ins}^*) = f_{is}^e.$$ (3)

Next, we consider the trade balance condition. From sectoral expenditure $R_{js} = \mu_s w_j \bar{L}_j$ in country $j$, denote the share of sectoral expenditure of country $j$ spent on goods from country $i$ by

$$\lambda_{ijs} \equiv \frac{R_{ijs}}{R_{js}} = \frac{R_{ijs}}{\mu_s w_j \bar{L}_j},$$

where $R_{ijs}$ is sectoral expenditure of country $j$ spent on goods from country $i$. Then the trade balance condition in country $i$ ($\sum_n \sum_s R_{nis} = \sum_n \sum_s R_{ins}$) is

$$w_i \bar{L}_i = \sum_{n=1}^N \sum_{s=1}^S \lambda_{ins} \mu_s w_n \bar{L}_n.$$ (4)

Finally, we look at the labor market clearing condition. In each sector $s \geq 1$, labor is used for entry and production ($L_{is} = M_{is}^e f_{is}^e + \sum_n L_{ins}$). Using (2), (3) and the definition of $V_{is}(\cdot)$ in section 2.1, the amount of labor used in sector $s$ of country $i$ is expressed as (see Appendix)

$$L_{is} = \frac{\sum_{n=1}^N R_{ins}}{w_i}.$$ 

The labor market clearing condition, aggregating the use of labor across sectors ($\bar{L}_i = \sum_s L_{is}$), is

$$\bar{L}_i = \frac{\sum_{n=1}^N \sum_{s=1}^S R_{ins}}{w_i}. $$
Noting that the first (second) subscript denotes the exporting (importing) country in this paper, aggregate expenditure in country $i$ consists of expenditure spent on domestic goods and foreign goods, $R_i = \sum_n \sum_s R_{nis}$, whereas aggregate labor income in country $i$ consists of revenues earned by domestic firms and exporting firms, $w_i \bar{L}_i = \sum_n \sum_s R_{nis}$. The labor market clearing condition is thus equivalent to the trade balance condition in that both conditions induce the same equality, $R_i = w_i \bar{L}_i$.

Now, we are ready for the characterization of the important variables in general equilibrium. For given exogenous variables, an equilibrium in levels can be defined as a set of \$\{\varphi_{ijs}^*, B_{is}, w_i\}$ which are jointly characterized by (2), (3), and (4) for $i, j = 1, \ldots, N$ and $s = 1, \ldots, S$. As is evident from the dependence of $s$, the productivity cutoffs $\varphi_{ijs}^*$ and the market demands $B_{is}$ are allowed to vary across sectors; in contrast, the wages $w_i$ are the same across sectors due to perfect intersectoral mobility of labor. This suggests that (2) and (3) are the two systems of equations that characterize $\varphi_{ijs}^*$ and $B_{is}$ in each sector, whereas (4) is the additional system of equations that characterize $w_i$ aggregating the use of labor across all sectors in each country.\footnote{To be precise, (3) holds in every sector so long as $M_{i}^e > 0$ (since $M_{i}^e = 0$ if the expected profits are smaller than the fixed entry cost). It can be easily shown that incomplete specialization arises when $L_i$ is not too different across countries.} In other words, the model has a recursive structure in that $\varphi_{ijs}^*$ and $B_{is}$ are independent of any sector aggregates (such as sector expenditure $R_{is}$ and sector labor supply $L_{is}$) and are analyzed separately from $w_i$, which is extensively used in monopolistic competition models with an outside good. Without loss of generality, we hereafter set $w_N = 1$ as a numeraire of the model in the absence of an outside good ($\mu_0 = 0$). In the presence of this good ($\mu_0 \neq 0$), on the other hand, $w_i = 1$ for all $i$, in which case the equilibrium is characterized by only (2) and (3), thereby omitting (4).

Once these key endogenous variables are determined, the other endogenous variables can be written as a function of the unknown variables. Using the definition of $B_{is}$ in (2) and rearranging, the real wage in country $i$ is determined by the domestic productivity cutoff $\varphi_{iis}^*$:

$$\frac{w_i}{P_{is}} = \frac{\sigma_s}{\sigma_s} \left( \frac{\mu_s \bar{L}_i}{\sigma_s \varphi_{iis}^*} \right)^{\frac{1}{\sigma_s - 1}} \varphi_{iis}^*.$$ 

Further, noting that the (nominal) wage $w_i$ is unity with an outside good, welfare per worker in our multi-sector model is given by

$$W_i = \begin{cases} \prod_{s=1}^S \left( \frac{w_i}{P_{is}} \right)^{\mu_s} & \text{if } \mu_0 = 0, \\ \prod_{s=0}^S \left( \frac{w_i}{P_{is}} \right)^{\mu_s} & \text{if } \mu_0 \neq 0. \end{cases}$$

It is clear that welfare per worker is proportional to the real wage in either case, and the impact of exogenous shocks on welfare can be inferred from changes in that wage. In the next sections, we investigate how changes in exogenous variables induce a different welfare effect, which varies critically with the existence of an outside good.
3 Trade Liberalization

In the previous section, we have defined the equilibrium conditions and the equilibrium variables in levels. In this section, we will define the equilibrium conditions and the equilibrium variables in changes. To highlight the role of wages in our model, we first examine the impact of changes in trade barriers, holding all other exogenous variables constant. This allows us not only to show how this impact crucially depends on an outside good, but also to better understand the impact of country size in the next section.

Demidova and Rodríguez-Clare (2013) investigate a welfare consequence of asymmetric trade liberalization in the Melitz (2003) model with two countries and one differentiated-good sector, dispensing with the assumption of an outside good. They show that reductions in trade barriers on either exports and imports increase welfare in a liberalizing country, which stands in sharp contrast to the presence of an outside good where reductions in trade barriers can reduce welfare in that country (Demidova, 2008; Melitz and Ottaviano, 2008). Here, with help of the exact hat algebra, we analytically show their result in our model. More importantly, we show in the next section that endogenous wages can reverse the impact of country size on productivity, just as in the impact of trade liberalization on welfare.

Let us consider the impact of variable trade costs. Suppose that country \( j \) unilaterally reduces variable trade costs on importing \( \tau_{ijs} \) from country \( i \) in sector \( s \). Denoting proportional changes of variables by a “hat” (i.e., \( \hat{x} = dx/x \)), and taking the log and differentiating the zero profit cutoff condition (2) with respect to \( \tau_{ijs} \),

\[
\hat{B}_{js} + (\sigma_s - 1)\hat{\varphi}_{ijs} = \sigma_s \hat{w}_i + (\sigma_s - 1)\hat{\tau}_{ijs}. \tag{5}
\]

Similarly, differentiating the free entry condition (3) with respect to \( \tau_{ijs} \),

\[
\sum_{n=1}^{N} f_{ins} J_{is} (\varphi_{ins}^s) \varphi_{ins}^s \hat{\varphi}_{ins}^s = 0. \tag{6}
\]

Finally, taking the log and differentiating the trade balance condition (4) with respect to \( \tau_{ijs} \),

\[
\hat{w}_i = \sum_{n=1}^{N} \sum_{s=1}^{S} \delta_{ins} (\lambda_{ins} + \hat{w}_n), \tag{7}
\]

where

\[
\delta_{ijs} \equiv \frac{R_{ijs}}{R_i} = \frac{\lambda_{ijs} w_j L_j}{\hat{w}_i L_i}.
\]

Just like (2), (3) and (4) can be used for the equilibrium in levels, (5), (6), and (7) can be used for the equilibrium in changes. More specifically, for given changes in variable trade costs \( \hat{\tau}_{ijs} \), the equilibrium in changes can be defined as a set of \( \{\hat{\varphi}_{ijs}, \hat{B}_{is}, \hat{w}_i\} \) which are jointly characterized by (5), (6), and (7) for \( i, j = 1, ..., N \) and \( s = 1, ..., S \). It follows immediately that the model has
also a recursive structure for the equilibrium in changes in that (5) and (6) are the two systems of equations that characterize \( \varphi_{ijs}^* \) and \( \dot{B}_{is} \) in each sector, whereas (7) is the additional system of equations that characterize \( \dot{w}_i \) in each country. Thus, in the absence of an outside good \( (\mu_0 = 0) \), we have that \( \dot{w}_N = 0 \), but in the presence of this good \( (\mu_0 \neq 0) \), we have that \( \dot{w}_i = 0 \) for all \( i \), in which case the equilibrium is characterized by only (5) and (6), thereby omitting (7).

Changes in the real wage are expressed as

\[
\dot{w}_i - \hat{P}_{is} = \varphi_{iis}^*,
\]

which shows that, to know what happens to welfare as a result of unilateral trade liberalization, we just need to see what happens to \( \varphi_{iis}^* \). The impact of trade liberalization on welfare, however, is quite different between endogenous wages \( (\mu_0 = 0) \) and exogenous wages \( (\mu_0 \neq 0) \) as long as all countries produce an outside good.

In order to provide the intuition for mechanisms in our model, let us consider a special case that is examined by Demidova and Rodríguez-Clare (2013). Assume that there are two countries \( (N = 2) \) and one differentiated-good sector \( (S = 1) \), and country 1 unilaterally reduces variable trade costs on importing \( \tau_{21} \) from country 2. Noting that \( \dot{w}_2 = 0 \) and dropping the sector subscript from all variables for notational simplicity, (5) gives us the four productivity cutoffs in changes:

\[
\begin{align*}
\dot{B}_1 + (\sigma - 1)\varphi_{11}^* &= \sigma \dot{w}_1, \\
\dot{B}_2 + (\sigma - 1)\varphi_{22}^* &= 0, \\
\dot{B}_2 + (\sigma - 1)\varphi_{12}^* &= \sigma \dot{w}_1, \\
\dot{B}_1 + (\sigma - 1)\varphi_{21}^* &= (\sigma - 1)\tau_{21}.
\end{align*}
\]

In this special case, (3) is rewritten as \( f_{11}J_{1}(\varphi_{11}^*) + f_{12}J_{1}(\varphi_{12}^*) = f_{1i}^* \) and \( f_{22}J_{2}(\varphi_{22}^*) + f_{21}J_{2}(\varphi_{21}^*) = f_{2i}^* \) for country 1 and country 2 respectively, and (6) gives us the relationship between the domestic productivity cutoff and export productivity cutoff in changes:

\[
\begin{align*}
\varphi_{12}^* &= -\alpha_1 \varphi_{11}^*, \\
\varphi_{21}^* &= -\alpha_2 \varphi_{22}^*.
\end{align*}
\]

where \( \alpha_i(>0) \) is a function of \( \varphi_{ii}^* \) and \( \varphi_{ij}^* \). Focusing on the trade balance condition in country 1 by Walras’s law, (4) is rewritten as \( w_1L_1 = \lambda_{11}w_1L_1 + \lambda_{12}w_2L_2 \), and (7) gives us the relationship between the wages and domestic productivity cutoffs in changes:

\[
\dot{w}_1 = -\beta_1 \varphi_{11}^* + \beta_2 \varphi_{22}^*,
\]

where \( \beta_i(>0) \) is a function of \( \varphi_{ii}^* \) and \( \varphi_{ij}^* \). This is a system of seven equations ((9), (10), (11)) with seven unknowns \( (\varphi_{11}^*, \varphi_{12}^*, \varphi_{12}^*, \varphi_{21}^*, \varphi_{22}^*, \dot{B}_1, \dot{B}_2, \dot{w}_1) \), but the equilibrium outcome is different between \( \mu_0 = 0 \) and \( \mu_0 \neq 0 \).
If $\mu_0 \neq 0$ so that wages are exogenously fixed by an outside good, we can solve (9) and (10) without referring to (11). This yields the following equilibrium relationships:

$$
\hat{\phi}_{11}^* = -\frac{\sigma(1 + \alpha_2)}{(\sigma - 1)(\alpha_1 \alpha_2 - 1)} \hat{w}_1 + \frac{1}{\alpha_1 \alpha_2 - 1} \hat{\tau}_{21},
$$

$$
\hat{\phi}_{22}^* = \frac{\sigma(1 + \alpha_1)}{(\sigma - 1)(\alpha_1 \alpha_2 - 1)} \hat{w}_1 - \frac{\alpha_2}{\alpha_1 \alpha_2 - 1} \hat{\tau}_{21},
$$

where $\alpha_1 \alpha_2 - 1 > 0$ (from the definition of $\alpha_i$). If wages are exogenous (i.e., $\hat{w}_1 = 0$), (12) shows that reductions in $\tau_{21}$ decrease $\phi_{11}^*$ but increase $\phi_{22}^*$, which in turn means that reductions in $\tau_{21}$ always reduce (raise) welfare in country 1 (country 2) due to a rise (a fall) in $P_1$ ($P_2$). This welfare result is in line with previous work with an outside good in which unilateral trade liberalization reduces welfare in a liberalizing country (Demidova, 2008; Melitz and Ottaviano, 2008).

If $\mu_0 = 0$ so that wages are endogenously determined by the trade balance condition, solving the system of equations (9), (10) and (11) simultaneously yields

$$
\hat{\phi}_{11}^* = -\frac{(\sigma - 1)[(\sigma - 1) + \sigma \beta_2]}{\Xi} \hat{\tau}_{21},
$$

$$
\hat{\phi}_{22}^* = -\frac{(\sigma - 1)[\sigma \beta_1 - (\sigma - 1) \alpha_1]}{\Xi} \hat{\tau}_{21},
$$

$$
\hat{w}_1 = \frac{(\sigma - 1)^2(\beta_1 + \alpha_1 \beta_2)}{\Xi} \hat{\tau}_{21},
$$

where $\Xi > 0$ and $\sigma \beta_1 - (\sigma - 1) \alpha_1 > 0$ (from the definitions of $\alpha_i$ and $\beta_i$). (13) shows that reductions in $\tau_{21}$ increase $\phi_{11}^*$, $\phi_{22}^*$ and decrease $w_1$. From (8), these changes in turn mean that welfare rises not only in country 2 but also in country 1 because a decline in $w_1$ is smaller than a decline in $P_1$ and hence $w_1/P_1$ rises.

The difference between (12) and (13) stems from the home market effect on trade patterns. Solving the price index $P_i$ for the mass of entrants $M_i^e$ yields

$$
\frac{M_1^e}{M_2^e} = \left(\frac{w_1}{w_2}\right)^{\sigma - 1} \frac{(P_1/P_2)^{1 - \sigma} V_2(\phi_{22}^*) - \tau_{21}^{1 - \sigma} V_2(\phi_{21}^*)}{V_1(\phi_{11}^*) - \tau_{12}^{1 - \sigma} (P_1/P_2)^{1 - \sigma} V_1(\phi_{12}^*)}.
$$

If $w_i$ is exogenous, applying (10) and (12) reveals that $M_1^e/M_2^e$ is decreasing in $\tau_{21}$, which means that trade liberalization in country 1 leads to redistributions of firms into the homogeneous-good (differentiated-good) sector in country 1 (country 2). Observing that $\phi_{12}^*$ rises whereas $\phi_{21}^*$ falls, the relative mass of exporting firms is decreasing in $\tau_{21}$. Further, firm export revenue satisfies

$$
\frac{r_{12}(\varphi)}{r_{21}(\varphi)} = \frac{B_2}{B_1} \left(\frac{\tau_{12} w_1}{\tau_{21} w_2}\right)^{1 - \sigma}.
$$

From (9) and (12), $r_{12}(\varphi)/r_{21}(\varphi)$ is also decreasing in $\tau_{21}$, which means that trade liberalization in country 1 changes the trade patterns in favor of country 2, not only through firm entry (extensive margin) but also through firm revenue (intensive margin). Intuitively, toughness of competition
remains unchanged by reductions in $\tau_{21}$ as both markups and price-cost margins are constant under C.E.S. preferences, and thereby such reductions affect only foreign market accessibility in the non-liberalizing country. As shown by Venables (1987), this results in welfare losses (gains) in the liberalizing (non-liberalizing) country. If $w_i$ is endogenous, in contrast, it follows from (13) that neither entry nor revenue is always decreasing in $\tau_{21}$, and hence the home market effect is not operative on the trade patterns. This is because reductions in $\tau_{21}$ decrease the wages $w_1$ and price-cost margins (1) even with C.E.S. preferences, which makes firms find it more difficult to earn domestic/export revenue in country 1. Since only more productive firms are able to survive forcing the least productive firms to exit there, both aggregate productivity and welfare rise in the liberalizing country.

Though we have focused on variable trade costs on imports $\tau_{21}$, the same holds for reductions in variable trade costs on exports $\tau_{12}$, as well as fixed trade costs $f_{21}$ and $f_{12}$. For example,

\[
\varphi^*_1 = - \frac{(\sigma - 1)[(\sigma - 1)\alpha_2 - (\sigma - 1)\alpha_2]}{\xi} \tau_{12},
\]

\[
\varphi^*_2 = - \frac{(\sigma - 1)[(\sigma - 1) + \sigma \beta_1]}{\xi} \tau_{12},
\]

\[
\hat{w}_1 = - \frac{(\sigma - 1)^2(\alpha_2 \beta_1 + \beta_2)}{\xi} \tau_{12}.
\]

From this, if $w_i$ is endogenous, reductions in $\tau_{12}$ raise the domestic productivity cutoffs in both countries. While welfare improves in a liberalizing country as above, reductions in export costs $\tau_{12}$ raise $w_1$ here, which implies that the intuition is different from import costs $\tau_{21}$. A rise in wages increases the price-cost margins and leads to decreased product market competition, but this also increases labor demand for exports and leads to increased factor market competition. The latter effect that makes the least productive firms unable to use labor dominates the former.

Together with the above finding, this shows that, if wages are endogenous, welfare increases for a country that unilaterally reduces trade barriers of both exports and imports.

The result is essentially the same as that in Demidova and Rodriguez-Clare (2013). They find that endogenous wages can reverse the impact of asymmetric trade liberalization on welfare in a liberalizing country due to a failure of the home market effect on trade patterns without an outside good. While they graphically show the finding with a simple figure, we analytically show a similar result with the exact hat algebra. A more important difference is however that while they focus on the impact of trade barriers, we can also examine the impact of another competitive measure, i.e., country size. To the best of our knowledge, no paper has investigated the impact of country size on aggregate productivity and welfare in the setting where country size has the home market effect on wages a la Krugman (1980).\textsuperscript{4} The exact hat algebra adopted here easily allows us not only to address this impact in a parallel manner with trade liberalization, but also to account for many of empirical facts demonstrated by non-C.E.S. models.

\textsuperscript{4}Demidova (2017) extends the Melitz-Ottaviano (2008) model to allow for endogenous wages by dispensing with an outside good, but her main focus lies on the impact of trade policy, not on the impact of country size in this paper.
4 Country Size

Let us next consider changes in country size, holding all the other exogenous variables constant, which has been extensively examined in the literature. Melitz and Ottaviano (2008) are the first to show that a country with larger size entails higher productivity and welfare through tougher competition in the domestic market, reducing firms’ average markups. Due to an outside good that gives rise to the home market effect on trade patterns, however, trade liberalization has an opposite impact from country size on welfare: a unilaterally liberalizing country can be worse off through the home market effect on trade patterns, which relocates production between countries (sometimes referred to as “firm delocation” in the literature).

We show that, in the absence of an outside good, endogenous wages can reverse the impact of country size, just as in the impact of trade liberalization: a country with larger size exhibits higher price-cost margins and less competitive pressures on firms by the home market effect on wages (Krugman, 1980), which stands in sharp contrast to previous work with an outside good. Though a country with larger size exhibits lower productivity (associated with higher price-cost margins), this country nevertheless enjoys higher welfare gains because a negative impact on declined productivity is always dominated by a positive impact on increased product variety.

Suppose that country $i$ unilaterally increases country size $L_i$, where $L_i$ satisfies $L_i = \sum_s L_{is}$ and is thus different from $L_{is}$. Denoting proportional changes of variables by a “hat” once again, and taking the log and differentiating (2) with respect to $\tilde{L}_i$,

$$\tilde{B}_{js} + (\sigma_s - 1)\tilde{\varphi}_{ijs} = \sigma_s \tilde{w}_i. \quad (14)$$

While (6) is the same as before, taking the log and differentiating (4) with respect to $\tilde{L}_i$,

$$\tilde{w}_i + \tilde{L}_i = \sum_{n=1}^N \sum_{s=1}^S \delta_{ins} (\tilde{\lambda}_{ins} + \tilde{w}_n) + \sum_{s=1}^S \delta_{iis} \tilde{L}_i. \quad (15)$$

The definition of the equilibrium in changes is similar as that in the previous section: for given changes in country size $\tilde{L}_i$, the equilibrium in changes can be defined as a set of $\{\tilde{\varphi}_{ijs}, \tilde{B}_{is}, \tilde{w}_i\}$ which are jointly characterized by (6), (14), and (15) for $i, j = 1, ..., N$ and $s = 1, ..., S$. Exploiting a recursive structure of the model, we have that $\tilde{w}_N = 0$ if $\mu_0 = 0$, while $\tilde{w}_i = 0$ for all $i$ if $\mu_0 \neq 0$ thereby omitting (15).

Changes in the real wage are given by

$$\tilde{w}_i - \tilde{P}_{is} = \tilde{\varphi}_{iis} + \frac{\tilde{L}_i}{\sigma_s - 1}, \quad (16)$$

which shows that, to know what happens to welfare as a result of unilateral population growth, we need to see what happens not only to $\tilde{\varphi}_{iis}$ but also to $\tilde{L}_i$. As in trade liberalization, the impact of country size on productivity and welfare depends critically on the existence of an outside good.
In order to build the intuition behind the model, let us reconsider the special case examined by Demidova and Rodríguez-Clare (2013) but we examine the impact of population growth that unilaterally takes place in country 1 here, keeping country size in a trading partner (country 2) constant. Then \((14)\) gives us the four productivity cutoffs in changes:

\[
\begin{align*}
\hat{B}_1 + (\sigma - 1)\hat{\varphi}_{11}^* &= \sigma \hat{w}_1, \\
\hat{B}_2 + (\sigma - 1)\hat{\varphi}_{22}^* &= 0, \\
\hat{B}_2 + (\sigma - 1)\hat{\varphi}_{12}^* &= \sigma \hat{w}_1, \\
\hat{B}_1 + (\sigma - 1)\hat{\varphi}_{21}^* &= 0.
\end{align*}
\]

(17)

While \((10)\) remains the same, \((15)\) is expressed as

\[
\hat{w}_1 + \hat{L}_1 = -\beta_1 \hat{\varphi}_{11}^* + \beta_2 \hat{\varphi}_{22}^*,
\]

(18)

where all variables appearing in the equilibrium in changes, including \(\beta_1\), are the same as those in the previous section. As before, \((10), (17),\) and \((18)\) are seven equations with seven unknowns, and the equilibrium outcome is different between \(\mu_0 = 0\) and \(\mu_0 \neq 0\).

If \(\mu_0 \neq 0\), we can solve \((10)\) and \((17)\) without referring to \((18)\):

\[
\begin{align*}
\hat{\varphi}_{11}^* &= -\frac{\sigma(1 + \alpha_2)}{(\sigma - 1)(\alpha_1\alpha_2 - 1)} \hat{w}_1, \\
\hat{\varphi}_{22}^* &= \frac{\sigma(1 + \alpha_1)}{(\sigma - 1)(\alpha_1\alpha_2 - 1)} \hat{w}_1.
\end{align*}
\]

(19)

(19) shows that if \(\hat{w}_1 = 0\), country size has no impact on the domestic productivity cutoff \((\hat{\varphi}_{11}^* = \hat{\varphi}_{22}^* = 0)\). From \((16)\), this in turn means that country size raises welfare in country 1 due only to increased product variety, as in the standard heterogeneous-firm model with C.E.S. preferences (e.g., Melitz, 2003), let alone the homogeneous-firm model (e.g., Krugman, 1980).

If \(\mu_0 = 0\), in contrast, solving \((10), (17),\) and \((18)\) simultaneously yields

\[
\begin{align*}
\hat{\varphi}_{11}^* &= -\frac{\sigma(\sigma - 1)(1 + \alpha_2)}{\Xi} \hat{L}_1, \\
\hat{\varphi}_{22}^* &= \frac{\sigma(\sigma - 1)(1 + \alpha_1)}{\Xi} \hat{L}_1, \\
\hat{w}_1 &= \frac{(\sigma - 1)^2(\alpha_1\alpha_2 - 1)}{\Xi} \hat{L}_1.
\end{align*}
\]

(20)

(20) shows that an increase in \(\hat{L}_1\) decreases \(\varphi_{11}^*\) but increases \(\varphi_{22}^*\), and \((16)\) means that welfare rises in country 2, whereas welfare can rise or fall in country 1, depending on the magnitude of a decline in \(\varphi_{11}^*\) (declined productivity) and a rise in \(\hat{L}_1\) (increased product variety).\(^5\)

It is important to stress that the negative impact on \(\varphi_{11}^*\) comes from the home market effect on \(w_1\) (i.e., a larger

\(^5\)The negative impact is absent in Krugman (1980) as productivity is exogenous.
country has higher wages) as in Krugman (1980). This causes the higher price-cost margins (1)
and less competitive pressures on firms, and hence makes it possible for less productive firms to
survive there. Note also that, in contrast to a non-C.E.S. models (Melitz and Ottaviano, 2008) in
which country size has no impact on the productivity cutoffs of a trading partner, country size
does affect these cutoffs in the present paper through the wages \( w_i \) that change competitiveness
across countries.

**Proposition 1**  *If wages are endogenous, we have:*

(i) *In increasing returns and monopolistic competition, a country with larger size entails higher
wages even under firm heterogeneity.*

(ii) *An increase in country size has a negative (positive) impact on the domestic productivity
cutoff in a growing (non-growing) country.*

The impact of country size on the wages and domestic productivity cutoffs accounts for many
of empirical facts even under C.E.S. preferences. On the one hand, the first finding on the wages
implies the variable pricing and price-cost margins that are higher in a richer country of higher
per capita income \( w_i \) (Alexandria and Kaboski, 2011; Fieler, 2011), which in turn leads to revenue
variation for the same variety across different export markets (Eaton et al., 2011).\(^6\) On the other
hand, the second finding on the domestic productivity cutoffs implies that less productive firms
enter (exit) in economic booms (recessions) with a rise (fall) in \( w_i \) (Nerard and Ramey, 2013).
The findings are in line with those in recent non-C.E.S. models (e.g., Bertoletti and Etro, 2017; Di
Comité et al., 2017), which cannot come without endogenous wages in standard C.E.S. models.

While endogenous wages can rationalize the empirical patterns demonstrated by non-C.E.S.
preferences, the mechanism behind the result is different between these non-C.E.S. models and
our C.E.S. model. For example, Bertoletti and Etro (2017) show that if consumers’ preferences
are represented by additively separable indirect utilities, national income generates the variable
markups. In the current paper, in contrast, even though consumers’ preferences are represented
by C.E.S. and the markups are constant, the price-cost margins are no longer constant because
per capita income \( w_i \) varies with country size.

As with trade liberalization, the difference between (19) and (20) stems from the home market
effect, where wages play an important role in determining trade patterns. From the labor market
clearing condition, the mass of entrants is alternatively expressed as

\[
\frac{M_1^e}{M_2^e} = \left( \frac{f_{22}(\varphi_{22}^*)^{-\sigma}V_2(\varphi_{22}^*) + f_{21}(\varphi_{21}^*)^{-\sigma}V_2(\varphi_{21}^*)}{f_{11}(\varphi_{11}^*)^{-\sigma}V_1(\varphi_{11}^*) + f_{12}(\varphi_{12}^*)^{-\sigma}V_1(\varphi_{12}^*)} \right) \frac{R_1}{R_2}.
\]

\(^6\)Following Schott (2008), it is also possible to interpret the prices as a signal of vertical differentiation in quality.
Under this interpretation, products that require relatively high wages are of high quality and firms sell products of
higher quality in a richer country of higher per capita income \( w_i \).
Furthermore, let $M_{ii} = [1 - G_i(\varphi_{ii}^*)]M_i^e$ and $M_{ij} = [1 - G_i(\varphi_{ij}^*)]M_i^e$ respectively denote the mass of domestic firms and that of exporting firms, which satisfy

$$\frac{M_{11}}{M_{22}} = \left(\frac{1 - G_1(\varphi_{11}^*)}{1 - G_2(\varphi_{22}^*)}\right) \frac{M_1^e}{M_2^e}, \quad \frac{M_{12}}{M_{21}} = \left(\frac{1 - G_1(\varphi_{12}^*)}{1 - G_2(\varphi_{21}^*)}\right) \frac{M_1^e}{M_2^e}.$$

If $w_i$ is exogenous, $R_i = L_i$ and country size has no impact on the values in the brackets above. This means that the mass of entrants increases proportionately to country size in the current single differentiated-good sector setting, and that both the mass of domestic firms and that of exporting firms increase proportionately to the mass of entrants. Population growth in country 1 thus gives rise to the following pattern of entry:

$$\frac{M_1^e}{M_2^e} = \frac{M_{11}}{M_{22}} = \frac{M_{12}}{M_{21}}.$$

From (14) and (19), we have that $r_{12}(\varphi)/r_{21}(\varphi)$ is not affected by country size. If $w_i$ is endogenous, in contrast, $R_i = w_i L_i$ and country size has an impact on the values in the brackets above. While the mass of entrants does not necessarily increase more than proportionately to country size and thereby the home market effect is not operative on trade patterns, the mass of domestic firms (exporting firms) increases more (less) than proportionately to the mass of entrants. Therefore, population growth in country 1 gives rise to the following pattern of entry:

$$\frac{M_{12}}{M_{21}} < \frac{M_1^e}{M_2^e} < \frac{M_{11}}{M_{22}}.$$

Further, $r_{12}(\varphi)/r_{21}(\varphi)$ is also decreasing in $L_i$, which means that population growth in country 1 changes the trade patterns in favor of country 2 through both extensive and intensive margins. Intuitively, country 1 with a relatively higher proportion of consumers has more incentive to save trade costs that are high enough to generate selection; thus firms find it less (more) profitable to export their products to a smaller (larger) country, allowing relatively less (more) exporting firms to exist in country 1 (country 2). Just like unilateral trade liberalization has an impact on welfare from the shift in trade patterns, unilateral population growth changes welfare from the shift in exporting firms from a growing country with higher wages to a non-growing country with lower wages, which gives the negative (positive) welfare impact in the growing (non-growing) country. As is originally shown by Bertoletti and Etro (2017), this shift in the trade patterns induced by country size can be thought of as business destruction (creation) where a richer (poorer) country with higher (lower) wages is characterized by concentration (expansion) of large exporting firms. In our C.E.S. model, the shift arises only when wages are endogenous.

Note, however, that the above entry pattern still holds even in this case.

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\[7\] In a multi-sector version of our model ($S > 1$), it can be shown that the mass of entrants increases more than proportionately to country size ($M_i^e/M_i^e > L_i/L_i$). As shown by Krugman (1980), this gives rise to the home market effect on trade patterns so that an increase in country size leads to disproportionately reallocations of labor to the differentiated-good sectors, allowing a larger country to enjoy higher welfare gains through increased product variety. Note, however, that the above entry pattern still holds even in this case.
**Proposition 2** If wages are endogenous, changes in country size induce a redistribution of firms and sales across countries so that a growing (non-growing) country reallocates resources relatively more to domestic (export) production.

The difference in (19) and (20) also yields empirically testable predictions on domestic/export shares that are different from those obtained in the standard monopolistic competition model with heterogeneous firms. First, let us consider the share of exporting firms on domestic firms in country \(i\) (which is less than unity under selection into the export market):

\[
\frac{M_{ij}}{M_{ii}} = \frac{1 - G_i(\phi_{ij}^*)}{1 - G_i(\phi_{ii}^*)}.
\]

Reflecting the fact that domestic firms (exporting firms) are relatively more (less) operative in a larger country, (20) shows that this share is decreasing in country size. The finding represents a sharp departure from that in the monopolistic competition model with an outside good, which shows that the share is either independent of country size under C.E.S. preferences, or increasing in it under quasi-linear-quadratic preferences (e.g., Melitz and Ottaviano, 2008) because \(\phi_{ii}^* (\phi_{ij}^*)\) is increasing (decreasing) in \(\bar{L}_i\). Second, the domestic share on total expenditure in country \(i\) is expressed in the present setup as (see Appendix)

\[
\lambda_{ii} = \frac{\alpha_i}{1 + \alpha_i}.
\]

From (20), this share is increasing in country size. Thus a country with larger size has a larger domestic share, as documented by a lot of empirical data (e.g., Eaton and Kortum, 2002). Though large domestic shares would typically encourage firms to export those products for which they have larger domestic markets (known as the Linder hypothesis), this is not the case in our model due to the feedback from country size on selection as seen above. With an outside good, however, the share is either independent of country size under C.E.S. preferences or decreasing in it under quasi-linear-quadratic preferences.

Ultimately, it is an empirical question to explore which predictions are more likely in reality. However, the channel through which country size has an impact through wages has been largely overlooked in the literature by introducing an outside good, despite substantial wage differences across countries. As found by Demidova and Rodríguez-Clare (2013), endogenous wages can offer a different lens through which to understand the real world especially when competitiveness of trading countries is endogenously adjusted by wages. Our main emphasis in this paper is that endogenous wages can fix the problem that arises under C.E.S. preferences, while preserving the usefulness of the workforce model in the new trade theory literature.

**Proposition 3** If wages are endogenous, the share of exporting firms is lower but the domestic share on total expenditure is higher in the larger country.
It remains to show the impact of country size on welfare in a growing country, which depends on the magnitude of a decline in $\varphi_{11}$ (declined productivity) and a rise in $\bar{L}_{1}$ (increased product variety). Applying (20) to (16), it is straightforward to confirm that the price index $P_{i}$ necessarily falls in both countries as a result of population growth in country 1, and thus welfare improves not only in country 2 but also in country 1. This means that the negative impact on productivity is always dominated by the positive impact on product variety. Hence, even though productivity is negatively affected by increased wages, our model features the welfare gains highlighted by Krugman (1980).

**Proposition 4** If wages are endogenous, a country that experiences population growth entails lower aggregate productivity but higher welfare.

5 Conclusion

This paper has presented a C.E.S. model of monopolistic competition with heterogeneous firms in which toughness of competition is endogenously adjusted by wages. We showed that endogenous wages give rise to the pricing and price-cost margins that vary across markets and countries, which in turn affect endogenous entry of firms that is not proportional to country size (or income). In this setting, we studied the impact of two competitiveness measures – trade liberalization and country size – on productivity and welfare. We found that while more competitive pressures by these two measures always lead to higher welfare, unilateral population growth has an opposite impact on productivity from unilateral trade liberalization through changes in wages and price-cost margins. Operating through this channel, our C.E.S. model yields a set of predictions that are consistent with recent non-C.E.S. models but cannot be obtained in standard C.E.S. models. Although rigorous empirical work exploring the channel is yet to come, we believe that our model can provide a different view of trade, while preserving the usefulness of the workforce model in the new trade theory literature.
A Appendix

A.1 Labor Market Clearing Condition

Labor supply in sector $s$ of country $i$ is

\[ L_{is} = M^e_{is} f^e_{is} + \sum_{n=1}^{N} L_{ins} \]

\[ = M^e_{is} f^e_{is} + M^e_{is} \sum_{n=1}^{N} \int_{\varphi_{ins}}^{\infty} l_{ins}(\varphi) dG_{is}(\varphi), \]

where $L_{ijs}$ is the amount of labor used for production in country $i$ for country $j$ in sector $s$ and

\[ l_{ijs}(\varphi) = f_{ijs} + \frac{\tau_{ijs} q_{ijs}(\varphi)}{\varphi}. \]

From the free entry condition in (3), the amount of labor used for entry $M^e_{is} f^e_{is}$ is expressed as

\[ M^e_{is} f^e_{is} = \frac{M^e_{is}}{w_i} \sum_{n=1}^{N} \left\{ \frac{1}{\sigma_s} \int_{\varphi_{ins}}^{\infty} r_{ins}(\varphi) dG_{is}(\varphi) \right. \]

\[ - \left. \left[ 1 - G_{is}(\varphi_{ins}) \right] w_i f_{ins} \right\}. \]

Regarding the amount of labor for production $\sum_n L_{ins}$, noting that

\[ l_{ijs}(\varphi) = f_{ijs} + \frac{\sigma_s - 1}{\sigma_s} \frac{r_{ijs}(\varphi)}{w_i}, \]

this is expressed as

\[ \sum_{n=1}^{N} L_{ins} = \frac{M^e_{is}}{w_i} \sum_{n=1}^{N} \left\{ \left[ 1 - G_{is}(\varphi_{ins}) \right] w_i f_{ins} + \frac{\sigma_s - 1}{\sigma_s} \int_{\varphi_{ins}}^{\infty} r_{ins}(\varphi) dG_{is}(\varphi) \right\}. \]

Summing up these terms, we obtain

\[ L_{is} = \frac{M^e_{is}}{w_i} \sum_{n=1}^{N} \int_{\varphi_{ins}}^{\infty} r_{ins}(\varphi) dG_{is}(\varphi) \]

\[ = \frac{\sum_{n=1}^{N} R_{ins}}{w_i}, \]

where, using (2), sectoral expenditure $R_{ijs}$ is expressed as

\[ R_{ijs} = M^e_{is} \sigma_s w_i f_{ijs}(\varphi_{ijs})^{1-\sigma_s} V_{is}(\varphi_{ijs}). \] (A.1)

Finally, summing labor supply in each sector, $\sum_s L_{is} = \sum_n \sum_s R_{ins} / w_i = \bar{L}_i$, which means that $R_i = w_i \bar{L}_i$. 

17
A.2 Properties of $\alpha_i$

We first show that

$$
\alpha_i = \frac{f_{ii} J'_i(\varphi_{ii}^*) \varphi_{ii}^*}{f_{ij} J'_i(\varphi_{ij}^*) \varphi_{ij}^*} = \frac{f_{ii} (\varphi_{ii}^*)^{1-\sigma} V_i(\varphi_{ii}^*)}{f_{ij} (\varphi_{ij}^*)^{1-\sigma} V_i(\varphi_{ij}^*)}.
$$

(A.2)

The definition of $\alpha_i$ follows immediately from differentiating (3) under the special case:

$$
f_{ii} J_i(\varphi_{ii}^*) + f_{ij} J_i(\varphi_{ij}^*) = f_i. 
$$

The equality in (A.2) follows from differentiating $J_i(\varphi^*)$ with respect to $\varphi^*$:

$$
J'_i(\varphi^*) = -\left(\frac{\sigma - 1}{\varphi^*}\right) [J_i(\varphi^*) + 1 - G_i(\varphi^*)]
= -((\sigma - 1)(\varphi^*)^{-\sigma} V_i(\varphi^*),
$$

where the second equality comes from noting that $J_i(\varphi^*) + 1 - G_i(\varphi^*) = (\varphi^*)^{1-\sigma} V_i(\varphi^*)$ from the definitions of $J_i(\cdot)$ and $V_i(\cdot)$. Substituting this equality into the definition of $\alpha_i$ gives us the result.

Next, we show several properties of $\alpha_i$.

- The first property is that $\alpha_i \alpha_j > 1$. To show this, from (2), we have that

$$
\left(\frac{\varphi_{ij}^*}{\varphi_{ii}^*}\right)^{\sigma-1} = \frac{\tau_{ij}^{\sigma-1} f_{ij} B_i}{f_{ii} B_j}.
$$

Using this and (A.2),

$$
\alpha_i \alpha_j = (\tau_{ij} \tau_{ji})^{\sigma-1} \left(\frac{V_i(\varphi_{ii}^*) V_j(\varphi_{jj}^*)}{V_i(\varphi_{ij}^*) V_j(\varphi_{ji}^*)}\right) > 1.
$$

The inequality follows from selection into the export market (i.e., $\varphi_{ij}^* > \varphi_{ii}^*$) and noting that $V_i(\varphi^*)$ is strictly decreasing in $\varphi^*$.

- The second property is that $\alpha_i = R_{ii}/R_{ij}$. The result follows from substituting (A.1) into the second equality of (A.2).

- The third property is that the expenditure shares, $\lambda_{ii}, \lambda_{ji}$, are written in terms of $\alpha_i$:

$$
\lambda_{ii} = \frac{R_{ii}}{R_{ii} + R_{ji}} = \frac{R_{ii}}{R_{ii} + R_{ij}} = \frac{\alpha_i}{1 + \alpha_i},
$$

$$
\lambda_{ji} = \frac{R_{ji}}{R_{ii} + R_{ji}} = \frac{R_{ij}}{R_{ii} + R_{ij}} = \frac{1}{1 + \alpha_i}.
$$

(A.3)

This follows from the second property and noting that (4) is expressed as $R_{ij} = R_{ji}$. 

18
A.3 Impact of Trade Liberalization

To show (11), note that (4) in country 1 under the special case is expressed as

\[ w_1 \tilde{L}_1 = \lambda_{11} w_1 \tilde{L}_1 + \lambda_{12} w_2 \tilde{L}_2. \]

Noting that \( w_2 = 1 \) and differentiating this with respect to \( \tau_{21} \),

\[ \dot{w}_1 = \frac{\lambda_{11}}{1 - \lambda_{11}} \dot{\lambda}_{11} + \dot{\lambda}_{12}. \] (A.4)

Differentiating (A.3) with respect to \( \tau_{21} \),

\[ \dot{\lambda}_{11} = \frac{1}{1 + \alpha_1} \dot{\alpha}_1, \quad \dot{\lambda}_{12} = -\frac{\alpha_2}{1 + \alpha_2} \dot{\alpha}_2. \]

Further, differentiating (A.2) with respect to \( \tau_{21} \) and using (10),

\[ \dot{\alpha}_i = -[\sigma - 1 + \gamma_{ii} + (\sigma - 1 + \gamma_{ij}) \alpha_i] \dot{\phi}_{ii}, \]

where

\[ \gamma_{ij} \equiv -\frac{d \ln V_i(\varphi_{ij})}{d \ln \varphi_{ij}^*} > 0. \]

Substituting these into (A.4) yields (11) where

\[ \beta_i \equiv \frac{\alpha_i}{1 + \alpha_i} [\sigma - 1 + \gamma_{ii} + (\sigma - 1 + \gamma_{ij}) \alpha_i]. \]  

Next, we show (13). From (9), (10), and (11), it follows that

\[ \dot{B}_1 + (\sigma - 1) \dot{\phi}_{11}^* = \sigma \dot{w}_1, \] (A.5)
\[ \dot{B}_2 + (\sigma - 1) \dot{\phi}_{22}^* = 0, \] (A.6)
\[ \dot{B}_2 + (\sigma - 1) \dot{\phi}_{12}^* = \sigma \dot{w}_1, \] (A.7)
\[ \dot{B}_1 + (\sigma - 1) \dot{\phi}_{21}^* = (\sigma - 1) \dot{\tau}_{21}, \] (A.8)
\[ \dot{\phi}_{12}^* = -\alpha_1 \dot{\phi}_{11}^*, \] (A.9)
\[ \dot{\phi}_{21}^* = -\alpha_2 \dot{\phi}_{22}^*, \] (A.10)
\[ \dot{w}_1 = -\beta_1 \dot{\phi}_{11}^* + \beta_2 \dot{\phi}_{22}^*. \] (A.11)

From (A.5), (A.7), (A.9), (A.11) and (A.6), (A.8), (A.10), (A.11) respectively,

\[ [(\sigma - 1) + \sigma \beta_1] \dot{\phi}_{11}^* - [\sigma \beta_2 - (\sigma - 1) \alpha_2] \dot{\phi}_{22}^* = - (\sigma - 1) \dot{\tau}_{21}, \] (A.12)
\[ -[\sigma \beta_1 - (\sigma - 1) \alpha_1] \dot{\phi}_{11}^* + [(\sigma - 1) + \sigma \beta_2] \dot{\phi}_{22}^* = 0, \]

Note that if \( G_i(\varphi) = 1 - (\varphi_{\min})^k \) in support with \([\varphi_{\min}, \infty)\), we have \( \gamma_{ii} = \gamma_{ij} = k - (\sigma - 1) \) and hence \( \beta_i = k \alpha_i \).
where
\[
\sigma \beta_i - (\sigma - 1) \alpha_i = \frac{\alpha_i}{1 + \alpha_i} \left[ (\sigma - 1)^2 (1 + \alpha_i) + \sigma (\gamma_{ii} + \gamma_{ij} \alpha_i) \right] > 0.
\]
Solving (A.12) for \( \phi_{11}^* \) and \( \phi_{22}^* \) and subsequently substituting them into (A.11) yields (13), where
\[
\Xi \equiv [ (\sigma - 1) + \sigma \beta_1 ] [ (\sigma - 1) + \sigma \beta_2 ] - [ \sigma \beta_1 - (\sigma - 1) \alpha_1 ] [ \sigma \beta_2 - (\sigma - 1) \alpha_2 ] > 0.
\]
Rearranging \( \Xi \), it is straightforward to show that the inequality always holds. Together with (9) and (10), these imply that
\[
\frac{d \phi_{11}^*}{d \tau_{21}} < 0, \quad \frac{d \phi_{22}^*}{d \tau_{21}} < 0, \quad \frac{d \phi_{12}^*}{d \tau_{21}} > 0, \quad \frac{d \phi_{21}^*}{d \tau_{21}} > 0, \quad \frac{dB_1}{d \tau_{21}} > 0, \quad \frac{dB_2}{d \tau_{21}} > 0, \quad \frac{dw_1}{d \tau_{21}} > 0.
\]
Further, from (8), we have that \( \frac{dP_1}{d \tau_{21}} > 0 \) and \( \frac{dP_2}{d \tau_{21}} > 0 \). In contrast, if \( w_i \) is exogenous,
\[
\frac{d \phi_{11}^*}{d \tau_{21}} > 0, \quad \frac{d \phi_{22}^*}{d \tau_{21}} < 0, \quad \frac{d \phi_{12}^*}{d \tau_{21}} < 0, \quad \frac{d \phi_{21}^*}{d \tau_{21}} > 0, \quad \frac{dB_1}{d \tau_{21}} < 0, \quad \frac{dB_2}{d \tau_{21}} > 0, \quad \frac{dw_1}{d \tau_{21}} = 0,
\]
and, from (8), we have that \( \frac{dP_1}{d \tau_{21}} < 0 \) and \( \frac{dP_2}{d \tau_{21}} > 0 \).

Following similar steps, we can derive the equilibrium in changes for \( \tau_{12} \):
\[
\begin{align*}
\phi_{11}^* &= -\frac{(\sigma - 1)[\sigma \beta_2 - (\sigma - 1) \alpha_2]}{\Xi} \tilde{\tau}_{12}, \\
\phi_{22}^* &= -\frac{(\sigma - 1)[(\sigma - 1) + \sigma \beta_1]}{\Xi} \tilde{\tau}_{12}, \\
\tilde{w}_1 &= -\frac{(\sigma - 1)^2 (\alpha_2 \beta_1 + \beta_2)}{\Xi} \tilde{\tau}_{12},
\end{align*}
\]
and those for \( f_{21} \):
\[
\begin{align*}
\phi_{11}^* &= -\frac{(\sigma - 1) + \sigma \beta_2}{\Xi} \tilde{f}_{21}, \\
\phi_{22}^* &= -\frac{\sigma \beta_1 - (\sigma - 1) \alpha_1}{\Xi} \tilde{f}_{21}, \\
\tilde{w}_1 &= \frac{(\sigma - 1)(\beta_1 + \alpha_1 \beta_2)}{\Xi} \tilde{f}_{21},
\end{align*}
\]
and those for \( f_{12} \):
\[
\begin{align*}
\phi_{11}^* &= -\frac{\sigma \beta_2 - (\sigma - 1) \alpha_2}{\Xi} \tilde{f}_{12}, \\
\phi_{22}^* &= -\frac{(\sigma - 1) + \sigma \beta_1}{\Xi} \tilde{f}_{12}, \\
\tilde{w}_1 &= -\frac{(\sigma - 1)(\beta_2 + \alpha_2 \beta_1)}{\Xi} \tilde{f}_{12}.
\end{align*}
\]
While reductions in trade costs on exports and imports raise \( \phi_{ii}^* \) and welfare in both countries, reductions in import costs \( \tau_{21}, f_{21} \) reduce \( w_1 \), while reductions in export costs \( \tau_{12}, f_{12} \) raise \( w_1 \).
A.4 Impact of Country Size

We first show (18). Differentiating (4) in country 1 under symmetry with respect to $\bar{L}_1$,

$$\dot{w}_1 + \dot{\bar{L}}_1 = \frac{\lambda_{11}}{1 - \lambda_{11}} \dot{\lambda}_{11} + \dot{\lambda}_{12},$$

whereas differentiating (A.3) with respect to $\bar{L}_1$ yields exactly the same expressions as above.

Next, we show (20). From (17) and (18),

$$\dot{B}_1 + (\sigma - 1)\dot{\phi}_{11}^* = \sigma \dot{w}_1, \quad (A.13)$$
$$\dot{B}_2 + (\sigma - 1)\dot{\phi}_{22}^* = 0, \quad (A.14)$$
$$\dot{B}_2 + (\sigma - 1)\dot{\phi}_{22}^* = \sigma \dot{w}_1, \quad (A.15)$$
$$\dot{B}_1 + (\sigma - 1)\dot{\phi}_{21}^* = 0, \quad (A.16)$$

$$\dot{w}_1 = -\beta_1 \dot{\phi}_{11}^* + \beta_2 \dot{\phi}_{22}^* - \dot{\bar{L}}_1. \quad (A.17)$$

From (A.9), (A.13), (A.15) and (A.10), (A.14), (A.16), (A.17) respectively,

$$[(\sigma - 1) + \sigma \beta_1] \dot{\phi}_{11}^* - [\sigma \beta_2 - (\sigma - 1)\alpha_2] \dot{\phi}_{22}^* = -\sigma \dot{\bar{L}}_1,$$

$$-\sigma \beta_1 - (\sigma - 1)\alpha_1 \dot{\phi}_{11}^* + [(\sigma - 1) + \sigma \beta_2] \dot{\phi}_{22}^* = \sigma \dot{\bar{L}}_1. \quad (A.18)$$

Solving (A.18) for $\dot{\phi}_{11}^*$ and $\dot{\phi}_{22}^*$ and substituting them into (A.17) yields (20).

Using (20), we show that welfare rises in a growing country. Substituting (20) into (16) yields

$$\ddot{P}_1 = -\frac{\sigma \dot{\bar{L}}_1}{2} [(1 + \alpha_1)(\beta_2 - (\sigma - 1)\alpha_2) + \beta_1(1 + \alpha_2)].$$

Noting that the values in the brackets are positive, the price index $P_1$ decreases with $\bar{L}_1$, which suffices to prove the result (as $w_1$ increases with it). Together with (10) and (17), we have

$$\frac{d\phi_{11}^*}{dL_1} < 0, \quad \frac{d\phi_{22}^*}{dL_1} > 0, \quad \frac{d\phi_{11}^*}{dL_1} > 0, \quad \frac{d\phi_{21}^*}{dL_1} < 0, \quad \frac{dB_1}{dL_1} > 0, \quad \frac{dB_2}{dL_1} < 0, \quad \frac{dw_1}{dL_1} > 0.$$

Further, from (16), we have that $dP_1/d\bar{L}_1 < 0$ and $dP_2/d\bar{L}_1 < 0$. In contrast, if $w_i$ is exogenous,

$$\frac{d\phi_{11}^*}{dL_1} = 0, \quad \frac{d\phi_{22}^*}{dL_1} = 0, \quad \frac{d\phi_{11}^*}{dL_1} = 0, \quad \frac{d\phi_{21}^*}{dL_1} = 0, \quad \frac{dB_1}{dL_1} = 0, \quad \frac{dB_2}{dL_1} = 0, \quad \frac{dw_1}{dL_1} = 0,$$

and, from (16), $dP_1/d\bar{L}_1 < 0$ and $dP_2/d\bar{L}_1 = 0$.

Finally, we show the impact of country size on $\alpha_i$ and $\lambda_{ii}$. From the previous derivation,

$$\frac{d\alpha_i}{dL_i} = -[\sigma - 1 + \gamma_{ii} + (\sigma - 1 + \gamma_{ij})\alpha_i] \frac{\alpha_i}{\phi_{ii}^*} \frac{d\phi_{ii}^*}{dL_i},$$

and, from (20), $\alpha_i$ is increasing in $\bar{L}_i$. Since $\lambda_{ii}$ is increasing in $\alpha_i$ from (A.3), we have $d\lambda_{ii}/d\bar{L}_i > 0$. 

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A.5 Mass of Entrants

We first show the mass of entrants in section 3. The price index is expressed as

$$ P_i^{1-\sigma} = \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \left\{ M_i^e w_i^{1-\sigma} V_i(\varphi_{ii}^*) + M_j^e (\tau_{ji} w_j)^{1-\sigma} V_j(\varphi_{ji}^*) \right\}. $$

Solving $P_i$ and $P_j$ for $M_i^e$ and $M_j^e$ and using the definition of $B_i$ and $B_j$, we obtain

$$ M_i^e = \frac{w_i^{\sigma-1} R_i^\sigma V_j(\varphi_{ji}^*) - \tau_{ji}^{1-\sigma} R_i^1 V_j(\varphi_{ji}^*)}{\Delta}, $$

where

$$ \Delta \equiv V_i(\varphi_{ii}^*) V_j(\varphi_{ji}^*) - (\tau_{ij} \tau_{ji})^{(1-\sigma)} V_i(\varphi_{ij}^*) V_j(\varphi_{ji}^*) > 0, $$

so long as $\varphi_{ij}^* > \varphi_{ii}^*$. Dividing $M_1^e$ by $M_2^e$ establishes the desired result. It follows from (A.19) that $M_1^e > 0$ and $M_2^e > 0$ if and only if

$$ \frac{1}{\tau_{12}^{\sigma-1}} \frac{V_1(\varphi_{12}^*) R_1}{V_1(\varphi_{11}^*) R_2} < \frac{B_1}{B_2} < \frac{\sigma-1}{1-2} \frac{V_2(\varphi_{22}^*) R_1}{V_2(\varphi_{21}^*) R_2}. $$

Noting that $B_1/B_2$ is proportional to $\bar{L}_1/\bar{L}_2$, this requires that country size is not too different between countries.

Next, we show the mass of entrants in section 4. Substituting (A.1) into $L_i = \frac{R_{ii} + R_{ij}}{w_i}$ and solving for $M_i^e$, the mass of entrants in (A.19) is alternatively expressed as

$$ M_i^e = \frac{R_i}{\sigma [f_{ii}(\varphi_{ii}^*)^{1-\sigma} V_i(\varphi_{ii}^*) + f_{ij}(\varphi_{ij}^*)^{1-\sigma} V_i(\varphi_{ij}^*)]}, $$

where we use the facts that $L_i = \bar{L}_i$ for $S = 1$ and $R_i = w_i \bar{L}_i$. Dividing $M_1^e$ by $M_2^e$ establishes the desired result.

Finally, we briefly show the extension to the multi-sector version of the model. In that case, the mass of entrants in each sector $M_s^e$ is proportional to (endogenously-determined) labor supply in each sector $L_i$. Using $w_i L_i = R_{ii} + R_{ij}$ and $R_{ij} = \lambda_{ij} \mu w_j \bar{L}_j$,

$$ \frac{L_1}{L_2} = \left( \frac{w_2}{w_1} \right) \left( \frac{\lambda_{11} w_1 \bar{L}_1 + \lambda_{12} w_2 \bar{L}_2}{\lambda_{22} w_2 \bar{L}_2 + \lambda_{21} w_1 \bar{L}_1} \right). $$

If $w_i$ is exogenous, $\lambda_{ij}$ is independent of $\bar{L}_i$ from (A.2) and (A.3). In addition, differentiating $L_1/L_2$ with respect to $\bar{L}_1$, this ratio is increasing and concave in $\bar{L}_1$ if and only if $\alpha_1 \alpha_2 > 1$. These suggest that, if country size is not too different across countries within (A.20), $L_1/L_2 > \bar{L}_1/\bar{L}_2$, so that an increase in country size leads to disproportionately reallocations of labor to the differentiated-good sectors. From the mass of entrants above, $M_1^e/M_2^e > \bar{L}_1/\bar{L}_2$, so that the mass of entrants increases more than proportionately to country size.
References


