Global Sourcing in Industry Equilibrium

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Abstract

This paper extends Antrás and Helpman’s (2004) model to a continuum of sectors to explore a full set of possibilities for organizational forms in global sourcing. We show that industries of the economy are divided into seven classes of sectors in terms of the prevalence of organizational forms. In particular, the coexistence of different organizational forms is less likely as the sectoral input intensity is more extreme, thereby letting intra-industry heterogeneity be less relevant to the firms’ organizational choices. It is also demonstrated that our equilibrium analysis offers a useful framework for examining comparative statics on the sectoral input intensity.

Keywords: Organizational forms, sectoral input intensity, intra-industry heterogeneity

JEL Classification Numbers: F12, F14

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1 Introduction

Fragmentation of production chains is one of the most prominent phenomena in a global economy. In particular, outsourcing has currently experienced rapid growth both within and across national borders, largely because the advance of information technology helps to lower costs to find suitable subcontractors around the world. While the growth of outsourcing in domestic and foreign production is frequently observed in every sector of any country, the extent to which firms engage in outsourcing is quite different across industries. For instance, within national borders, Bartel, Lach, and Sicherman (2010) find that there exists a significant variation in the percentage of outsourcing firms across 41 industries in Spain: it ranges from a low of 7.7% for man-made fibers to a high of 76% for agricultural and forest machinery. Similarly, across national borders, Defever and Toubal (2007) show that the share of arm’s length imports via foreign outsourcing in total French imports varies considerably across 14 industries from 50.8% for electrical component to 80% for wood and paper. The above evidence implies that some sector-specific characteristics might have an impact on the firm’s outsourcing decision domestically and internationally.

This paper develops a North-South trade model in which the firm’s organizational form for input acquisition is endogenously determined by sectoral characteristics. In our model, the production of final goods requires two specialized inputs provided by a final-good producer and a component supplier, and the producer in North insources or outsources the supplier’s input manufactured in either North or South. Since the producer has two control options (integration/outsourcing) and two locational choices (North/South), there are four possible organizational forms in global sourcing: domestic integration, domestic outsourcing, foreign integration (hereafter referred to as foreign direct investment or FDI), and foreign outsourcing. We define sectoral characteristics along the following two dimensions. First, each sector varies continuously with the input intensity (relative importance of the two distinct inputs to the final-good production), and this intensity has crucial impacts on the firm’s make-or-buy decision: vertical integration (resp. outsourcing) is more likely in a sector where the producer’s (resp. supplier’s) input is relatively more important (Antràs, 2003; Acemoglu, Aghion, Griffith, and Zilibotti, 2010). Second, each sector is also different in terms of the producer’s heterogeneity in productivity, and the higher degree of dispersion within a sector is associated with the greater likelihood of integration among offshoring firms (Helpman, Melitz, and Yeaple, 2004; Yeaple, 2006). The main purpose of this paper is to analyze how these two sector-specific factors jointly characterize the firm’s organizational form in equilibrium.

This issue of global sourcing is not entirely new in the literature, and was first addressed by the seminal work of Antràs and Helpman (2004). One of their limitations is that they focus primarily on

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1 Abramovsky and Griffith (2006) find that establishments using the Internet outsource about 10.6% more business services than those that do not in the UK, while Tomiura (2009) shows evidence suggesting that firms connected with computer networks tend to outsource some of manufacturing tasks in Japan.

2 This identification of foreign integration and FDI follows from Antràs and Helpman (2004).
the two sectors in an economy ("headquarter-intensive sector" and "component-intensive sector" in their terminology) and their prediction for the organizational forms is not widely applicable to the real world. The current paper aims at proposing a single unified framework for simultaneously analyzing sectors of the economy as a whole (including Antrás-Helpman’s sectors as a subset of ours), and provides an explicit condition under which each organizational form appears in industry equilibrium. Our key contribution is to show that industries are divided into seven classes of sectors in terms of the prevalence of organizational forms. More specifically, the coexistence of different organizational forms is less likely as the sectoral input intensity is more extreme, thereby letting intra-industry heterogeneity be less relevant to the firms’ organizational choices. Furthermore, the input acquisition via outsourcing and offshoring is more likely in a sector where the supplier’s input is more important. Since there are a variety of industries whose characteristic difference leads to a remarkable variation in the distribution of four organizational forms, this paper contributes to the literature by offering a systematic and comprehensive prediction across all industries in global sourcing.

To see the usefulness of our model, we examine comparative statics on the sectoral input intensity. We show that, if the relative importance of the producer’s and supplier’s input is subject to change over time, the environment in which the agents cannot sign enforceable contracts ex ante causes the holdup problem, and the firms’ control and locational choices of production gradually evolve to mitigate profit losses stemming from contract incompleteness. Consequently, the model displays a new channel through which contractual frictions can trigger fragmentation of production, not only for foreign sourcing but also for domestic sourcing. As will be clear from the analysis, whether the input intensity is increasing or decreasing through time is not essential for our key result and, more importantly, this application cannot be realized without the equilibrium analysis for a continuum of sectors in the economy, which is our major departure from the original Antrás-Helpman framework. The present paper also tries to connect the theoretical findings to several empirical studies, with a special emphasis on the relationship between the sectoral input intensity and organizational forms of heterogeneous firms. We confirm that the existing evidence is actually consistent with our model’s prediction, although there are still non-negligible gaps between this paper and econometric works.

The rest of the paper is organized as follows. Section 2 presents our setup, and Section 3 solves the model to derive the firms’ organizational forms in equilibrium. Building upon this equilibrium outcome, Section 4 examines comparative statics on the sectoral input intensity. Section 5 reviews the empirical evidence related to our theoretical prediction, and Section 6 concludes.

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3 We use the terms “industry” and “sector” synonymously in this paper.

4 Following the previous literature, we define “outsourcing” ("offshoring") as the procurement of inputs via domestic outsourcing or foreign outsourcing (via foreign outsourcing or FDI). For example, Abramovsky and Griffith (2006, pp. 595) define them as follows: “Outsourcing is the decision to make or buy, regardless of where the activity takes place. … Offshoring is about where the activity takes place, regardless of whether it is within the corporate boundary or outside it.”
2 Setup

Our approach draws heavily from the work of Antrás and Helpman (2004) and the model developed in this paper is a variant of their setup. Consider a world with two countries, North and South. There is a unit mass of identical consumers in the world with the following quasi-linear preferences:

\[ U = x_0 + \frac{1}{\mu} \int_0^1 X_j^\mu d\lambda, \quad 0 < \mu < 1, \]

where \( x_0 \) is consumption of a homogeneous good, and \( X_j \) is aggregate consumption of differentiated products \( x_j(i) \) of variety \( i \) in sector \( j \in (0,1) \), which is represented by a Dixit-Stiglitz C.E.S. function:

\[ X_j = \left[ \int_0^{n_j} x_j(i)^\alpha d\lambda \right]^{1/\alpha}, \quad \mu < \alpha < 1. \]

\( n_j \) denotes the number (measure) of varieties in sector \( j \), which is endogenously determined in equilibrium. \( \alpha > \mu \) implies that the elasticity of substitution between any two varieties in sector \( j \) is higher than that between \( x_0 \) and \( x_j(i) \). As is well-known, the above preferences generate the following inverse demand function:

\[ p_j(i) = X_j^{\mu-\alpha} x_j(i)^{\alpha-1}. \]  \hspace{1cm} (1)

Labor is the only factor of production for \( x_0 \) and \( x_j(i) \) and is inelastically supplied in a competitive labor market in each country. Wage rates in North and South are respectively denoted by \( w^N \) and \( w^S \), both of which are fixed with \( w^N > w^S \). In what follows, we focus on a particular sector and drop sector subscript \( j \) from all relevant variables.

The production of a differentiated good of variety \( i \), \( x(i) \), requires two distinct inputs, headquarter services \( h(i) \) and manufactured components \( m(i) \). These inputs are made with one unit of labor per unit of output, and the production technology is summarized in the following Hicks-neutral, Cobb-Douglas production function:

\[ x(i) = \varphi \zeta h(i)^z m(i)^{1-z}, \quad 0 < z < 1, \]  \hspace{1cm} (2)

where \( \zeta = z^{-z}(1-z)^{1-z} \). \( z \) measures the intensity of headquarter services and varies with sectors. It is immediately seen that headquarter services (manufactured components) are relatively more important for production in the sector where \( z \) is closer to one (zero). We associate the first (second) input with product development (simple assembly), which is offered by a final-good producer (a component supplier).

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5To do a consistent analysis, we assume that a representative agent consumes differentiated products of a continuum of sectors over the interval \((0,1)\). (As emphasized in Introduction, the sectors are discrete in Antrás and Helpman (2004).)

6As shown in Antrás and Helpman (2004), these assumptions on the wage rates can be justified in the general-equilibrium framework if: (i) the wage rate represents the labor productivity to produce \( x_0 \); and (ii) labor supply is large enough to produce \( x_0 \) in both countries.
For simplicity, South is assumed to be sufficiently less efficient at producing headquarter services relative to North so that this input is always produced in North, but manufactured components can be produced in either North or South. Both inputs are highly differentiated and relationship-specific in the sense that \( h(i) \) and \( m(i) \) have the higher value for particular variety \( x(i) \) (than other varieties \( x(i') \) for \( i' \neq i \)) and a third party (such as courts of law) cannot distinguish its true value. \( \varphi \) is a producer’s productivity level that differs among producers, and the larger \( \varphi \) is the more productive producer. We model these producers’ productivity differences in the spirit of Melitz (2003): Upon bearing a fixed cost of entry \( f_E \) (measured in Northern labor), producers receive their random draw \( \varphi \) from a known distribution function \( G(\varphi) \). After observing this productivity level, each producer decides whether to exit or not, and only a subset of producers are able to cover the fixed entry cost and start producing their variety.

If the producer of variety \( i \) enters the market, while producing headquarter services \( h(i) \) by itself, it decides whether variety-specific manufactured components \( m(i) \) are made in house by directly employing a supplier in its firm, or is outsourced to an independent supplier. In addition, the producer decides whether components \( m(i) \) are produced in North or South. We denote the former choice by \( k \in \{V, O\} \) (short for vertical integration or outsourcing) and the latter by \( \ell \in \{N, S\} \) (short for North or South). This means that each producer with a different \( \varphi \) chooses its organizational form among four options for procurement of \( m(i) \): \( (k, \ell) \in \{V, O\} \times \{N, S\} \). Unless there is ambiguity, we henceforth use the two terms “firm” and “(final-good) producer” interchangeably.

In any combination of \( (k, \ell) \), a contract is used for delivery of inputs and bargaining over the contract occurs between the producer and its supplier. We assume that they cannot write enforceable contracts ex ante as to the input level contingent on the revenue. Since both inputs \( h(i) \) and \( m(i) \) are relationship-specific for particular variety \( x(i) \), they renegotiate over the initial contract after production takes place. The ex post bargaining power of the producer depends critically on organizational form \( (k, \ell) \), because vertical integration entails the residual rights of control (Grossman and Hart, 1986), and North has the greater residual rights relative to South (Levchenko, 2007; Nunn, 2007). We formalize this idea in a way such that the residual rights affect the producer’s outside option when bargaining fails to agree in the renegotiation. In particular, under integration, the residual rights allow the producer to seize a fraction \( \delta^V \in (0, 1) \) (with \( \delta^N > \delta^S \)) of the output \( x(i) \) already produced in the renegotiation. This in turn allows the producer to seize a fraction \( (\delta^V)^\alpha \) of the revenue \( R(h(i), m(i)) \), which is thought of as the producer’s outside option.\(^7\) Under outsourcing, by contrast, the producer has no residual rights of control over the stand-alone supplier, and hence the producer’s outside option is zero irrespective of the supplier’s location. Provided that the revenue is distributed through symmetric Nash bargaining where the producer obtains

\^[7]From (1) and (2), the revenue is given by \( R(h(i), m(i)) = X^{\alpha - \alpha} \varphi^\alpha \zeta h(i)^\alpha m(i)^\alpha (1 - z) \). To make the model simple, the supplier’s outside option is normalized to be zero for any \( (k, \ell) \).
its outside option plus one-half of the quasi-rents,\(^8\) the producer’s ex post bargaining power \(\beta_k\) is

\[
\beta^N_k = \frac{1}{2}[1 + (\delta^N)^\alpha] > \beta^S_k
\]

\[
= \frac{1}{2}[1 + (\delta^S)^\alpha] > \beta^N_O = \beta^S_O = \frac{1}{2}.
\]

Either organizational form incurs the fixed organizational cost \(f^k\) (measured in Northern labor), which is associated with monitoring or communication activities between the producer and its supplier. With this interpretation, this fixed cost would be higher: (i) if the supplier is located in South \((f^S_k > f^N_k)\) since the producer is always in North; and (ii) if vertical integration is chosen \((f^V_k > f^O_k)\) since it creates inefficiencies due to “governance costs” (Williamson, 1985). Under the additional assumption \(f^S_O > f^N_V\), the fixed costs among the organizational forms are ranked as follows:

\[
f^S_V > f^S_O > f^N_V > f^N_O.\(^9\)
\]

The timing of events is composed of four periods. In period 1, the producer of variety \(i\) draws its productivity level \(\varphi\) from \(G(\varphi)\) and decides whether to exit the market. In period 2, the producer chooses its organizational form \((k, \ell)\) if it enters the market. In period 3, the producer and its supplier decide on the level of the relationship-specific inputs, \(h(i)\) and \(m(i)\), respectively. In period 4, final goods are sold to consumers and the ex post bargaining over the revenue takes place. The next section solves this model to derive the characteristics of industry equilibrium, such as the prevalence of firms’ organizational forms and the number of varieties produced in each sector. For notational simplicity, variety script \(i\) is suppressed below by letting it be understood that the following analysis applies for any producers.

### 3 Equilibrium

In period 4, final goods \(x\) are sold and the revenue \(R(h, m)\) is obtained. Then, \(R(h, m)\) is distributed to the producer with the share of \(\beta\) and to the supplier with share of \(1 - \beta\) in the ex post bargaining.

In period 3, the producer and its supplier choose their input level noncooperatively, taking \(\beta\) as given. In particular, contract incompleteness leads them to solve the following profit maximization problems:

\[
\max_h \pi^H_{hk} = \beta^\ell f^H_{hk} = \beta^\ell R(h, m) - w^N h + T - w^N f^H_{hk},
\]

\[
\max_m \pi^M_{mk} = (1 - \beta^\ell)R(h, m) - w^d m - T - w^N f^M_{mk},
\]

\(^8\)Antrás and Helpman (2004) consider generalized Nash bargaining where the producer obtains its outside option plus \(\beta \in (0, 1)\) of the quasi-rents. This simplification, however, does not qualitatively affect our main result.

\(^9\)This ordering is identical with that in Antrás and Helpman (2004). As they emphasize, the assumption \(f^V_k > f^O_k\) does not always hold but is more likely when managerial overload is more important than managerial economies of scope.
where \( f^\ell_{Hk} \) and \( f^\ell_{Mk} \) are respectively the fixed organizational cost that the producer and its supplier have to incur, with \( f^\ell_{Hk} + f^\ell_{Mk} = f^\ell_k \). \( T \) is a lump-sum transfer from the supplier to the producer, working to make the supplier break-even subsequent to the ex post bargaining in period 4. Taking account of the optimal input levels in period 3, the producer chooses its organizational form \((k, \ell)\) to maximize the joint profit \( \pi^\ell_k = \pi^\ell_{Hk} + \pi^\ell_{Mk} \) in period 2. From the first-order conditions, the optimal pricing rule and the total operating profits are respectively expressed as

\[
p^* = \left( \frac{1}{\varphi^\alpha} \right) \left( \frac{w^N}{\beta^\ell_k} \right)^z \left( \frac{w^f}{1 - \beta^\ell_k} \right)^{1-z},
\]

and

\[
\pi^*_k(\varphi, X, z) = X^{(\mu - \alpha)/(1 - \alpha)}\varphi^{\alpha/(1 - \alpha)}\Phi^\ell_k(z) - w^N f^\ell_k,
\]

where

\[
\Phi^\ell_k(z) = \frac{1 - \alpha}{p^\ast\varphi^{\alpha/(1 - \alpha)}}.
\]

From (4), we know that the variable profits are composed of: (i) the market demand \( X^{(\mu - \alpha)/(1 - \alpha)} \); (ii) the firm’s productivity level \( \varphi^{\alpha/(1 - \alpha)} \); and (iii) \( \Phi^\ell_k(z) \), whose economic interpretation is ambiguous.\(^\text{10}\)

To see this implication in more detail, let us first consider what happens if the agents were able to sign complete contracts. In such an environment, the input levels are ex ante verifiable and the agents could directly bargain over the operating profits \( \pi^\ast \) rather than the revenue \( R(h, m) \). Thus, the producer would choose \( h \) to maximize \( \pi^*_H = \beta^\ell_k \pi^* + T \) and the supplier would set \( m \) to maximize \( \pi^*_M = (1 - \beta^\ell_k) \pi^* - T \), where \( \pi^* = \pi^*_H + \pi^*_M \) is the joint profit in period 2 under complete contracting. Solving this problem gives the following first-order conditions:

\[
p^* = \left( \frac{1}{\varphi^\alpha} \right) \left( \frac{w^N}{\beta^\ell_k} \right)^z \left( \frac{w^f}{1 - \beta^\ell_k} \right)^{1-z},
\]

\[
\pi^*(\varphi, X, z) = X^{(\mu - \alpha)/(1 - \alpha)}\varphi^{\alpha/(1 - \alpha)}\Phi^\ast(z) - w^N f^\ell_k,
\]

where

\[
\Phi^\ast(z) = \frac{1 - \alpha}{(p^\ast\varphi^{\alpha/(1 - \alpha)})}.
\]

The comparison between complete and incomplete contracting suggests that \( \Phi^\ell_k(z) \) is a fraction of the variable profits that are related to the underinvestment (\textit{holdup}) problem. Evidently, the optimal pricing rule under incomplete contracting is \( \frac{1}{\beta^\ell_k (1 - \beta^\ell_k)^{1-z}} \) times higher than that under complete contracting, because the distortion created by incomplete contracting leads to the suboptimal input levels relative to complete contracting, which in turn leads to the lower variable profits \( \Phi^\ell_k(z) \) relative to \( \Phi^\ast(z) \). Given this

\(^{10}\)Note that \( \Phi^\ell_k(z) \) are independent of the firm’s productivity level because \( \varphi \) in the denominator is canceled out.
interpretation, $\Phi_{\ell}^k(z)$ must play a key role in determining the firms’ organizational forms since these forms are chosen to minimize the inefficiency stemming from the holdup problem (see, e.g., Antràs (2003)).

This is an economic interpretation of $\Phi_{\ell}^k(z)$ for a given sector. Our next interest is in determining how this variable differs across sectors. It is easy to verify that if the wage differential between the countries is large enough to satisfy

$$\frac{\beta_S}{1 - \beta_N} < \frac{w_N}{w_S},$$

(5)

$\Phi_{\ell}^k(z)$ has the following properties (see Appendix for proof):

$$\frac{\partial}{\partial z} \Phi_{V}^N(z) > 0, \quad \frac{\partial}{\partial z} \Phi_{O}^N(z) = 0, \quad \Phi_{V}^N(1) > \Phi_{O}^N(1), \quad \Phi_{V}^N(0) < \Phi_{O}^N(0),$$

$$\frac{\partial}{\partial z} \Phi_{V}^S(z) < 0, \quad \frac{\partial}{\partial z} \Phi_{O}^S(z) < 0, \quad \Phi_{V}^S(1) > \Phi_{O}^S(1), \quad \Phi_{V}^S(0) < \Phi_{O}^S(0).$$

(6)

Figure 1 illustrates the relationships among $\Phi_{\ell}^k(z)$ derived from (6). As the figure reveals, the ordering of $\Phi_{\ell}^k(z)$ significantly varies with the sectoral input intensity $z$. The intuition behind the figure is explained by Grossman and Hart’s (1986) celebrated proposition that, in the presence of incomplete contracts and relationship-specific investments, the residual rights of control should be allocated to the party whose contribution in a relationship is relatively more important in alleviating the holdup problem. This insight can directly apply to the current setup where the relative contribution of the two inputs is measured by the sectoral input intensity $z$. It implies that, regardless of its locational choice, the producer should choose vertical integration in the headquarter-intensive sectors where the producer’s input is more important,

11This is a sufficient condition under which $\Phi_{V}^S(z)$ becomes downward-sloping for $z \in (0, 1)$. There are arguably various patterns of $\Phi_{\ell}^k(z)$, depending on exogenous parameters such as the producer’s outside options $\delta'$ embedded into $\beta_{\ell}^k$, but our equilibrium analysis does not rely on these parameters. Indeed, even if (5) is not satisfied, we can get the organizational form in equilibrium by using the same reasoning developed below. We will focus on Figure 1 in this paper just for expositional purposes (and its relevance to our application examined in the next section).
while it should choose outsourcing in the component-intensive sectors where the supplier’s input is more important. Coupled with the interpretation of $\Phi'_f(z)$, this logic shows why $\Phi'_V(z)$ is greater (smaller) than $\Phi'_O(z)$ in the sectors of larger (smaller) $z$, thereby intersecting at only one point as in Figure 1. In other words, these intersections between $\Phi'_V(z)$ and $\Phi'_O(z)$ are the thresholds at which the tradeoff coming from the holdup problem between vertical integration and outsourcing is exactly offset in each location $\ell \in \{N,S\}$.

In period 2, the producer chooses control $k$ and location $\ell$ to maximize the profits (4) in equilibrium. Clearly, each firm chooses its organizational form so that

$$\pi(\varphi, X, z) = \max_{k \in \{V,O\}, \ell \in \{N,S\}} \pi_k^\ell(\varphi, X, z).$$

Note that the firm’s organizational form depends crucially on the sectoral input intensity $z$ through the holdup problem $\Phi'_k(z)$ as shown above. Because this effect on the organizational choice is quite different across sectors, let us first consider the class of sectors with $\tau < z < 1$, where the production uses headquarter services $h$ most intensively. In this case, Figure 1 shows that

$$\Phi'_V(z) > \Phi'_O(z) > \Phi'_O(z) > \Phi'_O(z).$$

In order to find the firm’s organizational form in this class of sectors, we draw the profit function (4) with slope $\Phi'_k(z)$ and intercept $-w^N f_k^N$ in the $(\varphi^{\alpha/(1-\alpha)}, \pi_k^\ell(\cdot))$ space. From (3) and (7), then, it follows that $\pi_k^N(\cdot)$ always lies under $\pi_k^O(\cdot)$ for any firm’s productivity level, indicating that Southern production never arises in the class of sectors. Moreover, since $\Phi'_V(z)$ is strictly increasing in $z$ and $\Phi'_O(z)$ is independent of $z$ from (6), there exists another threshold, namely $\tilde{z}$ ($> \tau$), above which all firms remaining in the sector attain the higher profits by vertical integration relative to outsourcing. Therefore, the model predicts that: (i) goods are manufactured only within vertically integrated firms in the most headquarter-intensive class of sectors ($\tilde{z} < z < 1$); and (ii) the most productive firms choose insourcing, whereas the less productive firms choose outsourcing in the next most headquarter-intensive class of sectors ($\tau < z < \tilde{z}$).

The production location of these classes of sectors occurs in the same country where product development takes place, i.e., North.

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12To see the economic reasoning behind the ranking of $\Phi'_k(z)$ in more detail, consider the sector of $z = 0$ where the supplier’s input is extremely important relative to the producer’s input. The figure shows that $\Phi'_O(z = 0) > \Phi'_V(z = 0) > \Phi'_O(z = 0)$, implying: (i) Southern production is more profitable than Northern’s irrespective of control $k$; and (ii) outsourcing is more profitable than vertical integration for given location $\ell$. The reason for (i) is that the producer can enhance profitability by exploiting a lower wage in South and hence $\Phi'_V(z = 0) > \Phi'_O(z = 0)$ for any $k$ (recall that $\Phi'_k(z)$ is a fraction of the variable profits). On the other hand, the reason for (ii) is that, since the supplier’s holdup problem is critical around $z = 0$, the producer can enhance profitability by outsourcing components $m$ to the supplier and hence $\Phi'_V(z = 0) > \Phi'_V(z = 0)$ for given $\ell$.

13Strictly speaking, this equilibrium arises only if the producer’s outside option in North $\delta^N$ is sufficiently large to satisfy $f_k^V / f_k^N < (1 + \delta^N)^{\alpha/(1-\alpha)}(1 + \delta^N - 2\delta^N (1 - z))$. 
Next, we move focus to the third headquarter-intensive class of sectors satisfying $\hat{z} < z < \bar{z}$. From Figure 1, the relationships among $\Phi^V_\ell(z)$ become

$$\Phi^S_V(z) > \Phi^N_V(z) > \Phi^S_O(z) > \Phi^N_O(z),$$

and, by the tradeoff between $\Phi^V_\ell(z)$ and $f^V_\ell$, FDI becomes the optimal organizational form for the most productive firms in the class of sectors. The reason why FDI arises in this medium headquarter-intensive class of sectors is as follows: since the production now requires more intensively the supplier’s input that can be manufactured cheaply in low-wage South, FDI allows firms to exploit the wage differential and to generate the higher variable profits than domestic sourcing, i.e., $\Phi^S_V(z) > \Phi^N_S(z)$. Due to Melitz’s (2003) type interaction between productivity differences and fixed locational costs, however, low-productivity firms are not able to undertake FDI in this class of sectors. Indeed, if FDI were chosen by these firms, the benefit from the lower wage could not outweigh the burden from the higher fixed cost, and the operating profits from FDI would be lower for them. As a result, they have no choice but to remain in North, despite the higher variable profits gained from FDI.

By the same reasoning, we have

$$\hat{z} < z < \bar{z} \iff \Phi^S_V(z) > \Phi^N_V(z) > \Phi^S_O(z) > \Phi^N_O(z),$$

$$\bar{z} < z < \hat{z} \iff \Phi^S_V(z) > \Phi^N_V(z) > \Phi^S_O(z) > \Phi^N_O(z),$$

$$0 < z < \bar{z} \iff \Phi^S_O(z) > \Phi^N_O(z) > \Phi^S_V(z) > \Phi^N_V(z).$$

Hence, it is possible that all four organizational forms coexist in (8a), whereas domestic integration disappears in (8b). In the least headquarter-intensive class of sectors in (8c), domestic outsourcing and foreign outsourcing remain and, if the wage differential between North and South is sufficiently large, there exists another threshold, namely $\tilde{z} (< \bar{z})$, below which only foreign outsourcing prevails in the class of sectors.

Figure 2 summarizes the organizational forms appearing in a subgame-perfect Nash equilibrium. In the seven classes of sectors, the organizational forms are in the order corresponding to the firms’ productivity levels. It is clear from the figure that the coexistence of different organizational forms is less likely as the sectoral input intensity is more extreme, thereby letting intra-industry heterogeneity be less relevant to the firms’ organizational choices. Moreover, input trade via outsourcing and offshoring

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14That is, $f^O_S/f^N_O < (w^N/w^S)^{(1-z)\alpha/(1-\alpha)}$.

15Antrás and Helpman (2004) focus mainly on the subset of sectors in Figure 2: “component-intensive sector” and “headquarter-intensive sector,” which respectively correspond with $\hat{z} < z < \bar{z}$ and $\bar{z} < z < \hat{z}$ in our model.

16Following Helpman et al. (2004) and Antrás and Helpman (2004), suppose that $G(\varphi)$ is a Pareto distribution:

$$G(\varphi) = 1 - \left(\frac{b}{\varphi}\right)^{\gamma}, \text{ for } \varphi \geq b > 0,$$
is more likely to arise in the sector where manufactured components $m$ are more important.

In period 1 where the producer draws $\varphi$ from $G(\varphi)$, the number of firms $n$ is endogenously determined by free entry. Since potential entrants are ex ante identical in the model, the free entry (FE) condition is defined as

$$\int_{\bar{\varphi}}^{\infty} \pi(\varphi, X, z) dG(\varphi) = w^N f_E,$$

which indicates that the expected profits of potential entrants must equal the fixed entry cost. $\bar{\varphi}$ denotes a productivity threshold at which the least-productive firm in the industry is indifferent about whether or not to exit. This threshold is determined by the zero cutoff profit (ZCP) condition:

$$\varphi = \inf \left\{ \varphi : \pi(\varphi, X, z) > 0 \right\}.$$

In equilibrium, the FE and ZCP conditions jointly determine implicit solutions for the productivity cutoff $\varphi$ and the sectoral consumption level $X$. Using these variables, we can also derive other variables of interest, such as the number of surviving firms, the number of potential entrants, and the relative prevalence of organizational forms in each of the seven classes of sectors.

For instance, consider the most headquarter-intensive class of sectors, $\tilde{z} < z < 1$, where only domestic integration prevails in equilibrium. By definition, $\pi(\varphi, X, z) = \pi^N_N(\varphi, X, z)$ and thus the ZCP condition is obtained by setting $\pi^N_N(\varphi_0, X, z) = 0$:

$$\varphi_0 = X^{(\alpha-\mu)/\alpha} \left[ \frac{w^N f^N_N}{\Phi^N_N(z)} \right]^{(1-\alpha)/\alpha}.$$

where $\gamma$ measures the dispersion of productivity within a sector. Under this specific parameterization, the above observation (i.e., intra-industry heterogeneity is less relevant to the firms’ organizational choices in the extreme input-intensity sectors) is equivalent to saying that the shape parameter $\gamma$ has fewer effects on the organizational choices as $z$ is closer to 0 or 1.
On the other hand, the FE condition is written as

\[ \Phi^E_N(z) [V(\infty) - V(\varphi)] / f_E + [1 - G(\varphi)] f_N = w^N X^{(\alpha - \mu)/(1 - \alpha)}, \]

where

\[ V(\varphi) = \int_0^\varphi y^{\alpha/(1 - \alpha)} \delta G(y). \]

Since the ZCP and FE conditions have two unknowns \((X, \varphi)\), these two equations jointly provide implicit solutions for these variables. The number of surviving firms \(n\) is then endogenously determined from these unknowns.¹⁷

## 4 Application

In this section, we demonstrate that our generalized framework of global sourcing has a wide range of applications to economic analysis. One of the intriguing possibilities of these applications is to examine comparative statics on the sectoral input intensity. To this end, following Antràs’ (2005) formulation, we assume that the contribution of headquarter services \(h\) to output (as measured by \(z\)) is inversely related to the maturity or age of a new product. As we will see later, this assumption on the inverse relationship is not crucial to the above purpose.

Our dynamic application to the static model is set as follows. Time is continuous with \(t \in [0, \infty)\) where infinitely-lived consumers’ preferences are captured by the inverse demand function (1) in any \(t\).¹⁸ In addition, although both the producer’s outside option \(\delta^F\) and the relative wage \(w^N/w^S\) are constant over time so that the relationships among \(\Phi^F_k(z)\) in Figure 1 are time-invariant, the input intensity of headquarter services \(h\) is assumed to be decreasing in time. That is, in contrast to the static model in which the whole sectors’ equilibria have been analyzed with exogenously fixed \(z\), our focus in the dynamic setup is on a particular sector whose input intensity is a function of time \(z = g(t)\) with

\[ g'(t) < 0, \quad g(0) = 1, \quad \text{and} \quad \lim_{t \to \infty} g(t) = 0. \] ¹⁹

Equation (9) tries to capture the standardization process of production in which a new commodity requires headquarter services \(h\) for product development in the early stages, but manufactured components \(m\) for simple assembly become significant inputs in production as the good matures. As Antràs (2005) stresses,

¹⁷Note that: (i) \(X\) and \(\varphi\) are a function of \(z\) since these are derived from the FE and ZCP conditions; and (ii) even with these conditions, the wage rates are still exogenous and the labor-market-clearing condition is required to endogenize \(w^\ell\).

¹⁸While the assumption that the consumers’ preferences are time-invariant follows from Antràs (2005), the inverse demand is slightly different from his formulation. This difference has impacts on \(X\) and \(\varphi\) (and thus on the number of varieties \(n\)) that are jointly determined by the ZCP and FE conditions. The organizational forms in equilibrium are not affected by this difference, however, because \(\Phi^F_k(z)\) is independent of these two endogenous variables.
the standardized nature of a new product, formalized by (9), is one of the premises in Vernon’s (1966) hypothesis. Finally, the organizational structure satisfying the subgame-perfect Nash equilibrium in the static game repeatedly arises in each period without any reputational concerns. This last condition would be ensured when the discount rate for future profits is sufficiently large.\(^\text{19}\)

Under the circumstance, the implications of comparative statics are straightforward. Since \(g(t)\) is a continuous and strictly decreasing function of \(t\), there exist the corresponding unique time-thresholds, which are ordered as follows:

\[
0 \leq g^{-1}(\bar{z}) < g^{-1}(\tau) < g^{-1}(\bar{z}) < g^{-1}(\bar{z}) < g^{-1}(\bar{z}) < \infty.
\]

This means that, by product maturity, the organizational transition evolves from \(z = 1\) to \(z = 0\) in Figure 2. Consequently, the model predicts that, as the input for product development (simple assembly) becomes relatively less (more) important in the production over time, the location gradually shifts from industrialized North to low-wage South, whereas the control gradually shifts from vertical integration to outsourcing.\(^\text{20}\)

It is important to note that incomplete contracts governing international transaction and the resulting holdup problems are key driving forces for the offshore transition, as originally argued by Antràs (2005). In contrast to Antràs (2005) who assumes Northern complete contracting that makes the boundaries of the firm there both indeterminate and irrelevant, our model additionally suggests that, if the agents in North cannot sign enforceable contracts ex ante and have to renegotiate over the initial contract ex post, this contractual friction should alter the firm’s make-or-buy decision even in the developed country. To see this, in the early stages of product standardization where the production takes place only in North \((0 \leq t < g^{-1}(\tau))\), it follows from (6) and (9) that

\[
\frac{\partial}{\partial z} \Phi_N^V(z) \cdot \frac{d}{dt} g(t) < 0, \quad \frac{\partial}{\partial z} \Phi_O^V(z) \cdot \frac{d}{dt} g(t) = 0,
\]

and hence, due to the supplier’s holdup problem, the marginal benefit of vertical integration relatively vanishes over time. This force coming from incomplete contracts and relationship-specific investments

\(^{19}\)We additionally assume that: (i) the firm’s productivity level does not change over time; and (ii) the FE condition is for the heterogeneous firm in the previous section, not for the representative firm in Antràs (2005, Appendix D).

\(^{20}\)If we derive the highest \(\Phi_k^\ell(z)\) across sectors in the economy, it directly follows from Figure 1 that

\[
\max_{k \in \{V,O\}, \ell \in \{N,S\}} \Phi_k^\ell(z) = \left\{ \begin{array}{ll}
\Phi_N^V(z) & \text{if } \tau < z < 1 \\
\Phi_N^V(z) & \text{if } \bar{z} < z < \tau \\
\Phi_S^O(z) & \text{if } 0 < z < \bar{z}
\end{array} \right..
\]

Accordingly, the organizational form by which to earn the highest variable profits is: (i) domestic integration, if \(\tau < z < 1\); (ii) FDI, if \(\bar{z} < z < \tau\); and (iii) foreign outsourcing, if \(0 < z < \bar{z}\). This implies that, if all firms are symmetric in terms of their productivity levels (\(\phi = 1\)) and the fixed costs are the same across organizational forms (\(f_k^\ell = f\)), Northern production occurs only within the firm boundaries in the dynamics (c.f., Antràs, 2005).
lets domestic outsourcing be optimal for profit-maximizing firms, thereby stimulating fragmentation of
the production process within national borders of North. In the later stages \((g^{-1}(\tau) < t < \infty)\), a similar
friction through international incomplete contracts further induce global production sharing by FDI and
foreign outsourcing because

\[
\frac{\partial}{\partial z} \Phi_S^Z(z) \cdot \frac{d}{dt} g(t) > \frac{\partial}{\partial z} \Phi_V^Z(z) \cdot \frac{d}{dt} g(t) > 0.
\]

Therefore, only when the product sufficiently matures, does offshoring become more profitable relative to
domestic sourcing, with foreign outsourcing enjoying relatively bigger benefits from standardization than
FDI. Notice that the input-intensity dynamics \((9)\) itself is not essential for this application. Evidently,
even if the input intensity for product development \(z\) is assumed to increase through time, our main
emphasis does not change: incomplete contracts can trigger fragmentation of the production process, not
only for foreign sourcing but also for domestic sourcing.

As a final remark, the firms’ entry and exit behavior must be emphasized in the above comparative
statics. While the number of surviving firms \(n\) fluctuates as the input intensity \(z\) declines,\(^{21}\) we cannot
say how it changes in the current setup because it strongly depends on the distribution function \(G(\varphi)\) and
the model yields only an implicit solution of \(n\). If we apply a specific parameterization to this distribution
(e.g., a Pareto distribution), it is possible to calculate an explicit solution of \(n\) to see the variation in the
number of surviving firms across different standardized stages.

5 Evidence

In this section, we briefly review the empirical evidence relevant to our prediction. In particular, we focus
on the relationship between the sectoral input intensity \(z\) and the organizational forms of heterogeneous
firms \((k, \ell)\) depicted in Figure 2.

Tomiura (2008) documents the empirical evidence on Antràs (2005) by using a firm-level dataset for
all manufacturing industries in Japan. He interprets the percentage of R&D expenditure in sales as a
measurement of the final-good producer’s input intensity \((z\) in our model), and compares the average R&D
intensity among Northern production, FDI, and foreign outsourcing. He finds that foreign outsourcing
is significantly least R&D intensive among the three organizational forms while FDI is of medium R&D
intensity (see Table 1 in Tomiura (2008)). This evidence is consistent with any range of \(z\) in Figure 2,
e.g., foreign outsourcing appears in relatively low \(z\) \((0 < z < \hat{z})\) whereas FDI emerges in intermediate \(z\)
\((\hat{z} < z < \tau)\) in the figure.

\(^{21}\)As argued in the end of Section 3, the number of varieties \(n\) is derived from \(X\) and \(\varphi\) that are endogenously determined
by the ZCP and FE conditions. Since these two conditions are a function of \(z\), we know that \(n\) (as well as \(X\) and \(\varphi\)) varies
with \(z\).
Acemoglu et al. (2010) make both theoretical and empirical comparisons between domestic integration and domestic outsourcing. Their theory predicts that the technology intensity of the producer \( z \) in our model is a crucial determinant of vertical integration. Identifying R&D intensity with the producer’s technology intensity \( z \), they find evidence that \( z \) is positively correlated with the likelihood of domestic integration, but negatively correlated with domestic outsourcing in plant-level data for the manufacturing sectors in the UK (see Tables 2 and 3 in Acemoglu et al. (2010)). Similarly, Tomiura (2009) makes an empirical comparison between domestic outsourcing and foreign outsourcing, interpreting R&D intensity as a proxy for the producer’s technology intensity as in Acemoglu et al. (2010) and Tomiura (2008).\(^{22}\) His main finding is that, in the manufacturing sectors in Japan, firms tend to prefer domestic outsourcing to foreign outsourcing when they are R&D intensive. He also finds that large-sized firms tend to choose foreign outsourcing (see Tables 2 and 3 in Tomiura (2009)). Notice that the empirical findings in Acemoglu et al. (2010) and Tomiura (2009) are, respectively, consistent with our theoretical prediction in \( \tilde{z} < z < 1 \) and \( 0 < z < \tilde{z} \) in Figure 2.

Defever and Toubal (2007) compare FDI and foreign outsourcing by undertaking an investigation into French firm-level data. Using various explanatory variables omitted in the previous empirical literature (e.g., contracting environment, bargaining power, etc), they find evidence that foreign outsourcing is more likely if the supplier’s input intensity (defined as the share of input from suppliers in total output; \( 1 - z \) in our model) is higher. Moreover, they confirm that the interaction term between the producer’s total factor productivity \( \varphi \) in our model) and the supplier’s input intensity is significantly positive, implying that more (less) productive firms tend to choose FDI (foreign outsourcing) when \( z \) declines in the current setup.\(^{23}\) These two empirical findings, both of which are statistically significant for alternative specifications, are consistent with our theory in \( \tilde{z} < z < \tilde{z} \) (see Tables 6-8 in Defever and Toubal (2007)).

Kohler and Smolka (2009) and Federico (2010) highlight the linkage between the firms’ productivity levels and four organizational forms in global sourcing à la Antràs and Helpman (2004). Using Spanish and Italian firm-level data, respectively, they find significant empirical support for Antràs-Helpman’s productivity sorting, e.g., FDI (domestic outsourcing) is chosen by the most (least) productive firms (see Tables 4-7 in Kohler and Smolka (2009) and Tables 5-9 in Federico (2010)). It can be said that these two papers are the closest empirical studies to our theoretical setup in that four organizational forms are simultaneously analyzed,\(^{24}\) however, their main focus is concentrated on the average productivity difference among four organizational forms across all sectors without paying adequate attention to the sectoral difference by controlling for the headquarter-intensity effect through industry-fixed-effects. If

\(^{22}\) Strictly speaking, the definition of R&D intensity is slightly different between them: Acemoglu et al. (2010) use the ratio of R&D expenditure to value added (calculated from a sample pre-dating their vertical integration measures), whereas Tomiura (2009) uses the percentage of R&D expenditure in sales as in Tomiura (2008).

\(^{23}\) Contrary to Antràs and Helpman (2004), Defever and Toubal (2007) assume that the fixed cost of foreign outsourcing is higher than that of FDI. As a consequence, the most productive firms choose foreign outsourcing in their estimation.

\(^{24}\) Tomiura (2007) also conducts the similar test, but the control problem for domestic firms is absent from his investigation.
Table 1. – Relationships between \( z \) and \((k, \ell)\) in the existing evidence

<table>
<thead>
<tr>
<th>Paper</th>
<th>Organizational form</th>
<th>Associated intensity in Figure 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tomiura (2008)</td>
<td>((k, N)^* &gt; (V, S) &gt; (O, S))</td>
<td>(0 &lt; z &lt; 1)</td>
</tr>
<tr>
<td>Tomiura (2009)</td>
<td>((O, N) &gt; (O, S))</td>
<td>(0 &lt; z &lt; \hat{z})</td>
</tr>
<tr>
<td>Acemoglu et al. (2010)</td>
<td>((V, N) &gt; (O, N))</td>
<td>(\hat{z} &lt; z &lt; 1)</td>
</tr>
<tr>
<td>Defever and Toubal (2007)</td>
<td>((V, S) &gt; (O, S))</td>
<td>(\tilde{z} &lt; z &lt; \hat{z})</td>
</tr>
</tbody>
</table>

\* Following Antrás (2005), Northern firm boundaries are assumed away in Tomiura (2008).

Note: Organizational forms are arranged by the strength of the producer’s input intensity \( z \). This \( z \) is measured by R&D intensity defined as \( \frac{\text{R&D expenditure}}{\text{sales}} \) in Tomiura (2008, 2009) and \( \frac{\text{R&D expenditure}}{\text{total value added}} \) in Acemoglu et al. (2010), while it is measured by \( 1 - \frac{\text{supplier’s input}}{\text{total output}} \) in Defever and Toubal (2007).

the empirical test were conducted for the cross-industry correlation between the input intensity and the share of organizational forms in different sectors, we expect that the result in Figure 2 would accord well with these possible tests that have been omitted in their works.

Table 1 summarizes these pieces of the econometric evidence. Although the results in the table are in keeping with our theory, few studies have examined the interaction between the firm’s four-way choice and the sectoral input intensity in the empirical literature. To the best of our knowledge, no available empirical evidence completely matches our theoretical prediction, largely due to the data limitation on firm-level activities.

6 Conclusion

This paper has explored how the sectoral input intensity and intra-industry heterogeneity jointly affect the firms’ organizational choices in global sourcing. The main distinction from the previous literature is in constructing a simple model that analyzes a continuum of sectors in the economy. We show that, under moderate conditions, industries are categorized into the seven classes of sectors in terms of the prevalence of organizational forms. It is also demonstrated that our equilibrium analysis offers a useful framework for examining comparative statics on the sectoral input intensity, helping to build a bridge between our theoretical prediction and the existing evidence.

Nonetheless, our theory is unsatisfactory from an empirical point of view and the current model has to incorporate further firm-level evidence to deliver richer and more realistic perspectives on a global economy. In particular, this paper has investigated the industry equilibrium in which every firm has only one organizational choice from four available options, but recent empirical studies point out that many productive firms are actually choosing multiple organizational forms in global sourcing. For instance, Kohler and Smolla (2009) find that nearly 80% of Spanish firms use complex strategies where several
sourcing channels are combined to procure several distinct inputs.\footnote{See also Defever and Toubal (2007), Federico (2010), and Tomiura (2007, 2009) for the similar evidence in France, Italy, and Japan, respectively. These studies additionally show that there exists a positive and statistically significant correlation between domestic outsourcing and foreign outsourcing in the input acquisition.} They also demonstrate that, if the multiple organizational choice is taken into account ("mutually inclusive" in their terminology), the productivity premia associated with the different sourcing strategies are no longer statistically significant. This evidence apparently suggests that we should develop a theoretical model in which the number of organizational forms for the input acquisition is endogenously determined. Given the rapid growth of vertical specialization over the past two decades, the role of multi-product (or multi-organizational) firms in intermediate-input trade is worth investigating.

This crucial feature in the organizational aspect is missing in the present paper, and more effort needs to be devoted to elaborating the model in this direction. We believe, however, that our theory sheds some new light on the existing knowledge of global sourcing, and the model’s prediction is able to bear future rigorous empirical tests.
Appendix

A.1 Proof of Equation (6)

It is immediate to show that $\Phi_V^N(1) > \Phi_V^O(1)$, $\Phi_V^O(0) < \Phi_V^O(0)$, $\Phi_V^S(1) > \Phi_V^S(1)$, and $\Phi_V^S(0) < \Phi_V^S(0)$. In this appendix, we focus on the derivation of $\frac{\partial}{\partial z} \Phi_k^N(z)$ for $k \in \{V, O\}$ and $\ell \in \{N, S\}$.

A.1.1 $\frac{\partial}{\partial z} \Phi_V^N(z) > 0$ and $\frac{\partial}{\partial z} \Phi_O^N(z) = 0$

Consider first $\frac{\partial}{\partial z} \Phi_O^N(z)$. From $\beta_O^N = \frac{1}{2}$ (symmetric Nash bargaining assumption), it follows that $\Phi_O^N(z)$ is independent of $z$, i.e., $\frac{\partial}{\partial z} \Phi_O^N(z) = 0$.

On the other hand, $\Phi_V^N(z)$ is written as

$$\Phi_V^N(z) = \frac{1 - \alpha [\beta_V^N z + (1 - \beta_V^N)(1 - z)]}{[(1/\alpha)(w^N/\beta_V^N)^{\alpha}(w^N/(1 - \beta_V^N))^{1-z}]^{1/(1-\alpha)}}. \quad (A.1)$$

Define

$$\rho(z) = 1 - \alpha [\beta_V^N z + (1 - \beta_V^N)(1 - z)] > 0,$$

and

$$\sigma(z) = \left[\left(\frac{1}{\alpha}\right) \left(\frac{w^N}{\beta_V^N}\right)^z \left(\frac{w^N}{1 - \beta_V^N}\right)^{1-z}\right]^{\alpha/(1-\alpha)} > 0.$$

That is, $\rho(z)$ and $\sigma(z)$ are the numerator and denominator of $(A.1)$, respectively. Differentiating $(A.1)$ with respect to $z$ yields

$$\frac{\partial}{\partial z} \Phi_V^N(z) = \frac{-\alpha(2\beta_V^N - 1) + \rho(z) \alpha \log \frac{\beta_V^N}{1 - \beta_V^N}}{\sigma(z)}. \quad (A.2)$$

Since $\beta_V^N > \frac{1}{2}$, $\rho(z)$ is decreasing in $z$:

$$\rho'(z) = -\alpha(2\beta_V^N - 1) < 0 \quad \text{if} \quad \beta_V^N > \frac{1}{2}.$$

Thus if the numerator of $(A.2)$ evaluated at $z = 1$ is positive, $\frac{\partial}{\partial z} \Phi_V^N(z) > 0$ holds. From the above definition, we have

$$\rho(1) = 1 - \alpha \beta_V^N,$$

and then the numerator of $(A.2)$ is given by

$$\xi(\beta_V^N) = -\alpha(2\beta_V^N - 1) + (1 - \alpha \beta_V^N) \frac{\alpha}{1 - \alpha} \log \frac{\beta_V^N}{1 - \beta_V^N}.$$ 

Then, we can easily see that

$$\xi\left(\frac{1}{2}\right) = 0, \quad \xi(1) = +\infty, \quad \text{and} \quad \xi'(\beta_V^N) \geq 0,$$

which mean that the numerator of $(A.2)$ is always positive for $\beta_V^N \in \left(\frac{1}{2}, 1\right)$. Therefore, as long as $\beta_V^N > \frac{1}{2}$, $\Phi_V^N(z)$ is monotonically increasing in $z$. \quad Q.E.D.

A.1.2 $\frac{\partial}{\partial z} \Phi_V^S(z) < 0$ and $\frac{\partial}{\partial z} \Phi_O^S(z) < 0$

First, we prove $\frac{\partial}{\partial z} \Phi_O^S(z) < 0$. From the text, $\Phi_O^S(z)$ is given by

$$\Phi_O^S(z) = \frac{1 - \alpha [\beta_O^S z + (1 - \beta_O^S)(1 - z)]}{[(1/\alpha)(w^S/\beta_O^S)^{\alpha}(w^S/(1 - \beta_O^S))^{1-z}]^{1/(1-\alpha)}}.$$
Noticing $\beta^S_{\alpha} = \frac{1}{2}$, we define the numerator and denominator of $\Phi^S_O(z)$ as follows:

$$\frac{\partial}{\partial z} \Phi^S_O(z) = \frac{-\hat{\rho}(z) \frac{w^N}{\hat{\sigma}(z)} \log \frac{w^N}{w^S}}{\alpha} < 0,$$

and

$$\alpha > 0.$$ 

Then, it follows from $w^N > w^S$ that $\Phi^S_O(z)$ is strictly decreasing in $z$:

$$\frac{\partial}{\partial z} \Phi^S_O(z) = \frac{-\hat{\rho}(z) \frac{w^N}{\hat{\sigma}(z)} \log \frac{w^N}{w^S}}{\alpha} < 0.$$

The proof of $\frac{\partial}{\partial z} \Phi^S_V(z) < 0$ follows similar steps. Rewrite $\Phi^S_V(z)$ as

$$\Phi^S_V(z) = \frac{1 - \alpha [\beta^S z + (1 - \beta^S)(1 - z)]}{\{(1/\alpha)(w^N/\beta^S)^z[w^S/(1 - \beta^S)]^{1-z}\}^{\alpha/(1-\alpha)}},$$

Let us define

$$\hat{\rho}(z) = 1 - \alpha [\beta^S z + (1 - \beta^S)(1 - z)] > 0,$$

and

$$\hat{\sigma}(z) = \left[ \frac{1}{\alpha} \right] \left( \frac{w^N}{\beta^S} \right)^z \left( \frac{w^S}{1 - \beta^S} \right)^{1-z} \right]^{\alpha/(1-\alpha)} > 0.$$

Then, we have

$$\frac{\partial}{\partial z} \Phi^S_V(z) = \frac{-\alpha (2\beta^S V - 1) + \hat{\rho}(z) \frac{\alpha}{1-\alpha} \left( \log \frac{\beta^S}{1 - \beta^S} - \log \frac{w^N}{w^S} \right)}{\hat{\sigma}(z)}.$$ 

Since $\beta^S > \frac{1}{2}$ and $\frac{\alpha}{1-\alpha} > 0$, the right-hand side of the above equation is negative for any parameter values only if

$$\frac{\beta^S}{1 - \beta^S} < \frac{w^N}{w^S}.$$ 

(5)

It is natural to assume that the relative wage is sufficiently large to satisfy (5) in the model, because the firm’s primary incentive to undertake Southern production is to exploit the wage differential between countries. Note that this incentive is of particular importance for the Antrás (2005) type product cycle, i.e., $\frac{\partial}{\partial \tau} \Phi^S_V(z) \cdot \frac{\partial}{\partial \tau} g(t) > 0$. 

\textit{Q.E.D.}

\textbf{A.2 Proof of the single-crossing property among $\Phi^S_V(z)$}

We show $\Phi^S_V(z)$ and $\Phi^S_O(z)$ intersect only once in this appendix, but the similar argument applies for different combinations of $\Phi^S_V(z)$. Details for proof of other combinations are available upon request.

From the text, we have

$$\Phi^S_V(z) = \frac{1 - \alpha [\beta z + (1 - \beta)(1 - z)]}{\{(1/\alpha)(w^N/\beta^S)^z[w^S/(1 - \beta^S)]^{1-z}\}^{\alpha/(1-\alpha)}}, \quad \Phi^S_O(z) = \frac{1 - \alpha}{\{(1/\alpha)(2w^N)^z(2w^S)^{1-z}\}^{\alpha/(1-\alpha)}},$$

where $\beta \equiv \beta^S_{\alpha}$ for notational simplicity. Setting $\Phi^S_V(z) = \Phi^S_O(z)$ and rewriting it gives

$$1 - \frac{1}{2} \alpha = [2\beta z (1 - \beta)^{1-z} \alpha/(1-\alpha)] \left(1 - \alpha [\beta z + (1 - \beta)(1 - z)]\right).$$

Clearly, $z$ in the above equality satisfies $z$ in Figure 1, and showing $z = z$ is unique is equivalent with showing $\Phi^S_V(z)$ and $\Phi^S_O(z)$ intersect at only one point $z$. Let the right-hand side of the above equation denote $f(z)$, which is strictly positive. Noting that the left-hand side is independent of $z$, it suffices for the uniqueness of $z$ to show that: (i) $f(0) < 1 - \frac{1}{2} \alpha$; (ii) $f(1) > 1 - \frac{1}{2} \alpha$; and (iii) $f'(z) > 0$. In what follows, we prove these three conditions in order.
(i) \( f(0) < 1 - \frac{1}{2} \alpha \): Let \( f(0) = [2(1 - \beta)]^{\alpha/(1 - \alpha)}|1 - \alpha(1 - \beta)| \equiv f(\beta) > 0 \), with \( f(\frac{1}{2}) = 1 - \frac{1}{2} \alpha \). Taking the logarithm of \( \hat{f}(\beta) \) and differentiating it with respect to \( \beta \), we get

\[
\frac{f'(\beta)}{f(\beta)} = \frac{\alpha}{1 - \alpha} \left[ \frac{\beta}{(1 - \beta)|1 - \alpha(1 - \beta)|} \right] < 0.
\]

Since \( \beta > \frac{1}{2} \), (i) holds.

(ii) \( f(1) > 1 - \frac{1}{2} \alpha \): Let \( f(1) = (2\beta)^{\alpha/(1 - \alpha)}(1 - \alpha\beta) \equiv \hat{f}(\beta) > 0 \). It follows from \( \hat{f}(\frac{1}{2}) = 1 - \frac{1}{2} \alpha \) and

\[
\frac{f'(\beta)}{f(\beta)} = \frac{\alpha}{1 - \alpha} \left[ \frac{1 - \beta}{\beta(1 - \alpha\beta)} \right] > 0
\]

that (ii) holds for \( \beta > \frac{1}{2} \).

(iii) \( f'(z) > 0 \): Taking the logarithm of \( f(z) \) and differentiating it with respect to \( z \), we get

\[
\frac{f'(z)}{f(z)} = \frac{\alpha}{1 - \alpha} \ln \frac{\beta}{1 - \beta} - \frac{\alpha(2\beta - 1)}{1 - \alpha[\beta_z + (1 - \beta)(1 - z)]}.
\]

Because \( \beta > \frac{1}{2} \), the first (second) term is positive (negative) in the right-hand side of the above. To show (iii), we consider the absolute maximum value of the second term for any \( \alpha, \beta, \) and \( z \), and the above equation is still positive. Regarding \( z \), it is easy to see that the second term is maximized at \( z = 1 \):

\[
\frac{f'(1)}{f(1)} = \frac{\alpha}{1 - \alpha} \left[ \ln \frac{\beta}{1 - \beta} - \frac{(1 - \alpha)(2\beta - 1)}{1 - \alpha\beta} \right].
\]

Next, we prove that the values in the brackets are necessarily positive for any \( \alpha \) and \( \beta \). It is evident that both terms in the brackets are zero when \( \beta = \frac{1}{2} \). In addition, the slopes of each term are respectively \( \frac{1}{\beta(1 - \beta)} \) and \( \frac{(1 - \alpha)(2 - \alpha)}{1 - \alpha\beta} \), and the former is always greater than the latter. To see this, note that the second slope is maximized at \( \alpha = 0 \) since it is strictly decreasing in \( \alpha \). Even in that case, we have \( \frac{1}{\beta(1 - \beta)} > \frac{(1 - 0)(2 - 0)}{(1 - 0)(1 - \alpha\beta)} = 2 \) for \( \beta > \frac{1}{2} \). These observations together suggest that the values in the bracket are strictly positive. This proves (iii).

\[Q.E.D.\]

### A.3 Simulation of Figure 1

To examine the simulation of Figure 1, we set parameter values as \( \alpha = 0.5, w^N = 1, w^S = 0.5, \delta^N = 0.3, \) and \( \delta^S = 0.05 \). These parameter values are chosen so that: (i) the producer’s outside option in North \( \delta^N \) is sufficiently large (footnote 13); and (ii) the relative wage between countries \( w^N/w^S \) is sufficiently large (footnotes 11 and 14). Under these conditions, we have \( \beta^N = \frac{1}{2}(1 + 0.3^0.5) = 0.77, \beta^S = \frac{1}{2}(1 + 0.05^0.5) = 0.61 \), and therefore

\[
\Phi^N(z) = \frac{1 - 0.5\cdot0.61z + 0.39(1 - z)}{(1 - 0.5^0.5z)\left(\frac{1}{\alpha}\right)^z};
\]

\[
\Phi^S(z) = \frac{1 - 0.5\cdot0.77z + 0.23(1 - z)}{(1 - 0.5^0.77z)\left(\frac{1}{\alpha}\right)^z};
\]

\[
\Phi^N(z) = \frac{1 - 0.5\cdot0.5}{(\frac{1}{\alpha})}\left(\frac{1}{\alpha}\right)^z; \quad \Phi^S(z) = \frac{1 - 0.5\cdot0.5}{(\frac{1}{\alpha})}\left(\frac{1}{\alpha}\right)^z.
\]

The following are the MATLAB codes and the generated figures. Figure A.1 is the case where the wage differential is sufficiently large so that (5) is satisfied (\( w^N = 1, w^S = 0.5 \)), while Figure A.2 is the case where it is violated (\( w^N = 1, w^S = 0.75 \)). As proved in Appendix A.1.2, \( \Phi^N(z) \) is no longer downward-sloping in the latter case. We exclude this possibility from the main analysis because of our vertical FDI assumption.
MATLAB code for Figure A.1

```matlab
a=[0:0.1:1]; % z
b=(1-0.5*(0.61*a+0.39*(1-a)))./(2*(1/0.61).^a.*(0.5/0.39).^(1-a)); % \Phi_V^S

c=(1-0.5*(0.77*a+0.23*(1-a)))./(2*(1/0.77).^a.*(1/0.23).^(1-a)); % \Phi_V^N

d=((1-0.5*0.5)/4)*ones(1,11); % \Phi_O^N

e=(1-0.5*0.5)./(2*(1/0.5).^a); % \Phi_O^S

plot(a,b,a,c,a,d,a,e);
xlabel('$$z$$', 'interpreter', 'latex');
ylabel('$$\Phi_k^\ell(z)$$', 'interpreter', 'latex');
legend('$\Phi_V^S(z)$','$\Phi_V^N(z)$','$\Phi_O^N(z)$','$\Phi_O^S(z)$');

h=legend;
set(h, 'interpreter', 'latex');
```

MATLAB code for Figure A.2

```matlab
a=[0:0.1:1]; % z
b=(1-0.5*(0.61*a+0.39*(1-a)))./(2*(1/0.61).^a.*(0.75/0.39).^(1-a)); % \Phi_V^S

c=(1-0.5*(0.77*a+0.23*(1-a)))./(2*(1/0.77).^a.*(1/0.23).^(1-a)); % \Phi_V^N

d=((1-0.5*0.5)/4)*ones(1,11); % \Phi_O^N

e=(1-0.5*0.5)./(2*(1/0.5).^a.*(0.75/0.5).^(1-a)); % \Phi_O^S

plot(a,b,a,c,a,d,a,e);
xlabel('$$z$$', 'interpreter', 'latex');
ylabel('$$\Phi_k^\ell(z)$$', 'interpreter', 'latex');
legend('$\Phi_V^S(z)$','$\Phi_V^N(z)$','$\Phi_O^N(z)$','$\Phi_O^S(z)$');

h=legend;
set(h, 'interpreter', 'latex');
```

**Figure A.1.** – In case of $w^S = 0.5$

**Figure A.2.** – In case of $w^S = 0.75$
References


